Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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в	г	е	ё	ж	и	й	у	х	ц	ч	ш	щ	ъ	ы	ь	э	ю	я
v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	"	ĭ	'	è	yu	ya

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April 1950 | Vol. 3, No. 4

Communications

Concerning the query about a fracture-stress formula, December 1949.

Concerning stresses in ring-shaped bodies, we would like to call attention to the paper by E. A. Ripperger and N. Davis ["Critical stresses in a circular ring," Proc. Am. Soc. civ. Eng. 72, 159–168 (1946)], the discussions on pp. 1185–1190 and 1291–1294 of the same volume and on p. 531 of vol. 73. The following formula for the maximum stress is given: $s_{max} = KP/\pi r_0$, where r_0 is the outer radius of the ring, P is the applied load per unit thickness of the ring, and K is a factor dependent on $r' = r_i/r_0$, the ratio of the inner radius to the outer radius of the ring.

This formula was used by the writers to calculate fracture stresses in plaster rings for values of r ranging from 0.1875 to 0.750. Results were in excellent agreement with those obtained from beams of comparable size loaded at the mid-point.

Bach's formula,

$$s_{max} = 2(F/s)(xy - x + y)/x(1 + x)(1 - y),$$

is in wide discrepancy with that given by Ripperger and Davis. The discrepancy ranges from 90% greater for $r = 0.2$ to 60% smaller for $r = 0.8$ approximately, with an almost linear variation in-between. The two formulas give identical results for $r = 0.54$.

A. J. Durelli and R. Jacobson

Concerning Rev. 3, 381 (Feb. 1950), "Lateral earth pressure as a problem of deformation or of rupture," by G. P. Tschebotarioff and P. P. Brown.

This review was excessively delayed. The up-to-date state of the research is presented in a paper covered by an earlier review, 2, 1460 (Nov. 1949).

Ed.

Theoretical and Experimental Methods

(See also Revs. 628, 629, 637, 650, 685, 701, 740, 751)

594. **E. Bodewig, A report on various methods of solution of a system of linear equations with real coefficients I, II, III** (in German), Nederl. Akad. Wetensch. Proc. 50, 930–941, 1104–1116, 1285–1295 = Indagationes Math. 9, 441–452, 518–530, 611–621 (1947).

After a general discussion of the difficulties involved in the solution of simultaneous equations in many unknowns the author proceeds to an analysis of the number of operations required by various methods of solution. He concludes that for n unknowns: (1) Gauss's method requires $\frac{1}{3}\, n(n^2 - 1) + n^2$ multiplications, $\frac{1}{6}\, n(n - 1)(2n + 5)$ additions; (2) Cholesky's method (applicable only to symmetric systems) requires n square roots, $\frac{1}{6}\, n(n^2 + 9n + 2)$ multiplications, $\frac{1}{6}\, n(n - 1)(n + 7)$ additions; (3) by Schur's method the calculation of the inverse matrix requires n^3 multiplications, $n^3 - 2n^2 + 2n$ additions; the solution of the system of equations requires in addition n^2 multiplications, $n^2 - n$ additions.

Parts II and III continue this analysis for the various methods associated with the names of Boltz, Banachiewicz, Jossa, Schmidt, von Mises, Pollaczek-Geiringer, Seidel, Morris; the relaxation method of Southwell, the iteration method of Cesari, etc. Part I deals with direct solutions; part II, with methods that first find the inverse matrix; part III, with methods of iteration.

Each method presented is accompanied by a commentary and comparison with other similar methods; for instance: "The most practical of all methods is a modification of Gauss's method in which Gauss's algorithm is used iteratively." The method of Cholesky is applicable only to symmetric equations and is longer than that of Gauss but has the advantage of requiring only a small amount of written work. "If one has to solve k sets of n equations with the same matrix, Gauss's method is better than Schur's if $k < n/3$. For $k > n/3$ the reverse is true." The widespread opinion that Schmidt's method of orthogonalization is the most advantageous is shown to be false. "The normalization of a set of equations [in preparation for solution by iteration] requires for itself alone $1\frac{1}{2}$ times as many operations as the complete solution by Gauss's method. The normalization by the method of von Mises is hence utterly uneconomical, since after its completion one still has the solution of the new system to perform." "The relaxation method of Southwell is identical with the method of Seidel." Comparing the speed of convergence of the ordinary iteration, that of von Mises, and that of Seidel, he says, "The method of von Mises is the worst and that of Seidel the best."

Courtesy of Mathematical Reviews W. E. Milne, USA

595. **Board of Trade Journal, Research in industry. A series of articles,** London, His Majesty's Stationery Office, 1948, 84 pp. 9.5 × 6 in., \$0.21.

A short introduction on the purposes and functions of the British Department of Scientific and Industrial Research is followed by two- to five-page summaries of the research activities of associated research organizations for trade associations in the fields of various materials and products. These are of considerable interest to those interested in research on a trade-association basis, particularly handcraft or craftmanship industries. These articles point out the difficulties and limitations of cooperative applied research and cite a few examples of successful accomplishments under English conditions.

K. W. Miller, USA

596. **John T. Pettit, A speedy solution of the cubic,** Math. Mag. 21, 94–98 (1947).

The equation is reduced to $y^3 + py - q = 0$ or $y^3 - py + q = 0$, where $p, q > 0$, and this by $y = qz/p$ to $z^3/(1 - z) = \pm K$, where $K = p^3/q^2$. The paper gives a graph for (K, z) and a table with the interval 0.01 for the root Z.

Courtesy of Mathematical Reviews E. Bodewig, Holland

597. **H. S. Wall, A modification of Newton's method,** Amer. Math. Monthly 55, 90–94 (1948).

The author rediscovers the formula $x_1 = x_0 - f/(f' - A)$, where $A = ff''/2f'$ and all functions are taken at the starting value $x = x_0$. Calling generally a sequence with the limit X

"convergent of degree n" if $\lim [(x_{p+1} - X)/(x_p - X)^n] = c \neq 0$, it can easily be proved that for an analytic f the above formula gives a sequence of convergence 3, if the root to be approximated is simple, and of convergence 1, if the root is multiple, whereas Newton's original formula gives sequences of convergence 2 and 1, respectively. A similar formula giving convergence of degree n can easily be constructed.
Courtesy of Mathematical Reviews E. Bodewig, Holland

598. P. Buchner, Horner's method for complex function values (in German), Elem. Math. 3, 8–11 (1948).

This is, as the author states, an exposition of Collatz's well-known extension of Horner's scheme for complex arguments [Z. angew. Math. Mech. 20, 235–236 (1940)]. Neither the author nor Collatz mentions the outstanding feature of Horner's scheme, namely the reduction of the labor of calculation which, compared with the usual labor, is merely $\frac{1}{2}$ in the case of a real and even $\frac{1}{3}$ in the case of a complex argument. When calculating the Newton correction $-f(a)/f'(a)$ the author commits the technical error of applying Horner's scheme to $f'(x)$ for computing $f'(a)$ instead of deriving $f'(a)$ from the continuation of Horner's scheme for $f(x)$ in a manner similar to that for a real argument.
Courtesy of Mathematical Reviews E. Bodewig, Holland

599. E. Franckx, Integration of systems of differential equations and the method of successive approximations (in French), Bull. Soc. Sci. Liége, 281–286 (July-Oct. 1948).

The author discusses the Picard, Ceressia, and the Ceressia-Germay method of obtaining the numerical solution of a normal system of differential equations and shows that the iterations may be performed in an arbitrary order. He then demonstrates that the functions obtained by such a process converge uniformly toward the particular solution of the system of normal equations.
Henry J. Barten, USA

600. Nils Zeilon, Some problems of numerical accuracy in the theory of differential and integral equations, Acta Univ. Lundensis Sect. 2, 43, no. 10 = Acta Reg. Soc. Physiog. Lund. 58, no. 10, 30 pp. (1947).

In the first part of the paper the variational problem of the asymptotic phase of $u'' + \rho(x)u = 0$ is treated by approximating the differential equation itself. For the two cases of $\rho(x) = k^2 + \rho_1(x)$, namely when $\rho_1(x) \sim ax^{-2}$ is a potential of Yukawa or similar type, and $\rho_1(x) = -L(L+1)x^{-2} + be^{-x}x^{-1}$, i.e., the equation of the deuteron, the degree of accuracy attained is determined. The second part deals with the numerical computation of the characteristic numbers λ_1 of an integral equation by iteration, thus avoiding linear equations. A method, similar to that of Trefftz, is described by which a very precise determination of λ_1 is obtained when a very rough limitation of $\lambda_2 > \lambda_1$ is available. The method is that of approximating the first characteristic function of the problem.
Courtesy of Mathematical Reviews E. Bodewig, Holland

601. R. H. Germay, Remark on the use of linear differential equations with variable coefficients in the method of integration of normal differential equations by successive approximation (in French), Bull. Soc. Roy. Sci. Liége 18, 3–8 (Jan. 1949).

The author points out that when the right-hand side of the equation $dz/dx = F(z, x)$ has the form $a(x)z + G(z, x)$, better results may be obtained from the method of successive approximations applied in the fashion

$$dz_{n+1}/dx = a(x)z_{n+1} + G(z_n, x)$$

than from the method applied in the straightforward manner. This idea is well known and forms an essential part of the stability theory of nonlinear differential equations. R. Bellman, USA

602. Udo Wegner, A remark on the characteristic values of matrices (in French), C. R. Acad. Sci. Paris 228, p. 1200 (Apr. 4, 1949).

603. François H. Raymond, An observation on stability in relation with matrix characteristic values (in French), C. R. Acad. Sci. Paris 228, 1564–1565 (May 16, 1949).

In the first paper, a method is given which makes it possible to determine whether the complex matrix $A = a_{ik}$ has eigenvalues whose real part is positive (stability criterion) without finding the roots of the characteristic equation. The transformation $\omega = (\lambda + 1)/(\lambda - 1)$ transforms A into $B = (A - E)^{-1}(A + E)$. If the largest eigenvalue of B, to be found by iteration, has a modulus less than unity, all eigenvalues of A have negative real parts.

The second paper adds the remark that all eigenvalues of A have positive or negative real parts, according as the elements of $(A + E)^{-1}(A - E)$ or B, respectively, are of moduli less than $1/n$ (n being the order of A). E. F. Masur, USA

604. Lothar Collatz, Eigenvalue problems and their numerical treatment (Eigenwertprobleme und ihre numerische Behandlung) New York, Chelsea Publishing Company, 1948, 338 pp., 104 figs. Cloth, 8.75 × 5.25 in., $4.50.

This book deals primarily with approximate methods for solving problems in elastic stability and vibration known as characteristic-value problems. They may be expressed as ordinary, linear, homogeneous differential equations of the form $M[y] = \lambda N[y]$, subject to certain linear boundary conditions, where λ is the characteristic value representing the buckling load or natural frequency, and M and N are linear, homogeneous differential expressions of the type $\Sigma(-1)^n[f_n(x)y^{(n)}(x)]^{(n)}$. The theory is developed principally from the differential-equation approach, although some space is devoted to integral equations and variational methods. The usual undergraduate courses in mathematics required of engineers will prove sufficient for an understanding of most of the book; where more extensive knowledge is necessary, the theory is developed in more detail. The book contains much of the original work of E. Kamke and the author in the field of the ordinary differential equations of characteristic-value problems.

The first chapter contains a discussion of several familiar problems of stability and vibration. An extensive table of important characteristic-value problems is given; for each problem there is tabulated the differential equation, boundary conditions and a reference where the solution may be found. The book is amply supplied throughout with problems and useful tables which express its practical nature.

The mathematical concepts of self-adjointness, general orthogonality and definiteness are presented in chapter 2. Green's function and the method for its construction for both ordinary and partial differential equations is introduced. The final section consists of some results from the theory of integral equations.

In chapter 3 the mathematical treatment is extended to consider the characteristic-value problem as a variational problem in order to show the minimum property of the characteristic values. The important "straddling" theorem, which establishes an upper and lower bound for the characteristic value when applying the Stodola iteration procedure, is presented. A discussion is given of the validity of the expansion of a function which satisfies the boundary conditions, but not necessarily the differential equation, in a series of the characteristic functions of the problem; the theory is not developed for the most general type of problem under consideration.

Chapter 4 gives a method similar to that of Rayleigh for the approximate determination of the characteristic values, with the important distinction that the accuracy may be successively improved by iteration, exactly as in the Stodola procedure. The method provides a means for obtaining upper and lower bounds for the characteristic value; this procedure has been employed in a recent paper [C. C. Miesse, Determination of the buckling load for columns of variable stiffness, J. appl. Mech. 16, no. 4, 406–410 (Dec. 1949)]. The application of graphical integration to the method is also given.

The variational method is used in chapter 5 to discuss the Ritz, Galerkin and Grammel procedures. In chapter 6 the method of finite differences is applied to the solution of characteristic-value problems. Chapter 7 presents the perturbation method and the Dunkerley and Southwell formulas. The book closes with a convenient survey of the examples treated.

Robert P. Felgar, Jr., USA

605. L. W. Pollak and C. Heilfron, Harmonic analysis and synthesis schedules for three to one hundred equidistant values of empiric functions, Dublin: Department of Industry and Commerce, Geophysical Publications, vol. 11, 1947, xxxiii + 118 pp. Approx. \$9.

The chief part of the work consists of tables for facilitating the computation of the first coefficients p_1, q_1 of the Fourier series, namely tables for cos iz and sin iz, where $z = 2\pi/n$, $i = 0,1,\ldots,n-1$ for $n = 3,4,\ldots,100$ at five decimals. Then according to Bessel, $p_1 = (2/n)\ \Sigma u_i \cos iz$, $q_1 = (2/n)\ \Sigma u_i \sin iz$, where u_i are the equidistant empirical values. Other tables, such as for values of iz, follow. The introduction contains Bessel's formulas for the Fourier coefficients, the division of the ordinates into groups, Lamont's correction for noncyclic change and Weihrauch's formulas.

Courtesy of Mathematical Reviews E. Bodewig, Holland

606. Pierre Humbert and Serge Colombo, Symbolic calculus and its applications to mathematical physics (in French), Mémor. Sci. Math. no. 105, 52 pp. (1947).

In this monograph, the authors have collected together the important theorems concerning the Carson and Bromwich-Wagner integrals with applications of these theorems to the properties of six functions and to circuit analysis. The material presented is not new but does appear in a useful condensed form.

The paper consists of an introduction and three parts. The introduction presents some operational concepts and theorems due to Heaviside. Aside from the historical fact that the Carson integral arose from a critical study of Heaviside methods, a presentation of Heaviside's methods adds little or nothing to the main body of the text.

Twenty basic theorems are derived in part I. These include the theorems on linearity, scale change, complex differentiation, real differentiation, real integration, real translation, complex translation, complex integration, and complex multiplication. These theorems are followed by the derivation of transform pairs for t^n, ϵ^{at}, sin ωt, cos ωt, sinh ωt, cosh ωt, and ln$t$. Part I closes with a derivation of the Bromwich-Wagner integral from Cauchy's theorem and Carson's integral.

The results of part I are used in part II to establish the fundamental identities and transform pairs associated with Bessel functions, the ber and bei functions of Kelvin, the logarithmic integral and the related sine and cosine integrals, the integrals of Fresnel and Gilbert, the functions $v(t) = {}_0\int^\infty [t^s/\Gamma(s+1)]ds$, and periodic functions.

In part III the authors have applied the methods of Carson to the steady state and transient solutions of linear electric circuits. The solution by operational methods of a set of linear differential equations is described. Detailed applications are made to the cases of two and three meshes magnetically coupled. The treatise closes with a solution of the wave equation by operational methods.

Although there is little new material in this paper, it can serve as a useful condensation of the most important theorems resulting from the Carson and Bromwich-Wagner integrals. The methods of Laplace are not treated at all.

Horace M. Trent, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 602, 603, 635, 636, 637, 701)

607. René Garnier, A course in kinematics (Cours de cinématique), vol. I, Paris, Gauthier-Villars, 1949, 235 pp., 64 figs. Paper, 10 × 6 25 in.

The first edition of this volume appeared in 1940 in printed form, while the present second edition is lithographic. It contains some changes and has been equipped with an index. Vectors, the moving trihedron and infinitesimal considerations are used. Chapter 2 ends with the study of the field of accelerations of a solid.

The work is remarkable for its elegant exposition, precise definitions and rigorous proofs. Volume 2 of this work appeared in 1941, and volume 3 is announced. Ratip Berker, Turkey

608. Gino Goldoni, On the acceleration center in the motion of a free rigid body (in Italian), Atti Sem. Mat. Fis. Univ. Modena 1, 12–16 (1947).

609. Rosenauer, N. On the construction of velocities of kinematic chains and mechanisms, Contrib. Baltic Univ. no. 19, 5 pp. (1947).

The known theorem that, for a carried line, the projections of the velocities of different points of the line on the line are equal, enables a graphical process which is applied to the derivation of the velocities of linkages employing one or two ternary links. (A ternary link is one which is joined to three other links at three distinct points.) The paper is essentially the same as an earlier paper by the author in German [Z. angew. Math. Mech. 17, 173–176 (1937)]. M. Goldberg, USA

610. Rosenauer, N. On the construction of accelerations of kinematic chains and mechanisms, Contrib. Baltic Univ. no. 32, 10 pp. (1947).

The acceleration of a point A in a link is equal to the acceleration of the point B in the same link plus the acceleration of A around B. This equation is resolved into its tangent and normal components in accordance with graphical methods described by Grübler and Wittenbauer. These methods are applied to the linkages of the foregoing review. Related methods have been described and referenced by Federhofer [*Graphische Kinematik und Kinetostatik*, Springer, Berlin, 1932, pp. 20–22].

M. Goldberg, USA

611. Rosenauer, N. On the construction of accelerations of kinematic chains and mechanisms including slide couples in movable planes, Contrib. Baltic Univ. no. 38, 14 pp. (1947).

The following theorem is demonstrated and applied to several linkage mechanisms which have slide couples in movable members. If in a chain or mechanism two links are connected through a slide couple, the representations of two points form a parallelogram, two sides of which are the relative normal ac-

celerations of these points and the other two are the Coriolis accelerations. The extremities of the different accelerations of these points form a second parallelogram with sides which are mutually perpendicular to the sides of the first. Two of these sides are the slide accelerations and the other two are relative tangential accelerations of these points. The construction is simpler than one given by Beyer in *Technische Kinematik* [Barth, Leipzig, 1931]. M. Goldberg, USA

612. Gaston Fleischel, A generalization of the Willis formula for epicyclic gear trains (in French), C. R. Acad. Sci. Paris 226, 220–222 (1948).

The three ratios of the angular speeds of the three members of an epicyclic gear train are proportional to p, $(p-1)/p$, $1/(1-p)$. This relation holds equally as well for any three gears in more complicated gear trains. M. Goldberg, USA

613. A. J. Staring, On central motions, in particular that along an ellipse (in Dutch), Simon Stevin 25, 208–223 (1947).

The author gives a general formula for the acceleration of a central motion, including the case that the center is at infinity. He considers, in particular, central motions along a conic and deduces the law of force when the orbit is an ellipse. An apparatus is described for the demonstration of the motion of a planet and of the components of a double star.

Courtesy of Mathematical Reviews O. Bottema, Holland

614. Z. Jankovich, The cycloid as a tautochrone and a brachistochrone (in Croatian, with English summary), Hrvatsko Prir. Dr. Glasnik Mat. Fiz. Astr. 2, 49–72 (1947).

615. E. A. Trabant, The Riemannian geometry of the symmetric top, Revista Ci. Lima 49, 269–281 (1947).

The Riemannian space described by a symmetric gyroscope is investigated. As a necessary and sufficient condition for the configuration space to be an Einstein space with constant curvature, it is found that the two moments of inertia computed with respect to the principal axes should be equal. G. Kron, USA

616. Mario Manarini, On some notable properties of the center of inertia in rigid body rotation (in Italian), Boll. Un. mat. ital. 3, 214–219 (Dec. 1948).

A solid body rotates uniformly around a fixed axis Δ; if the forces of inertia are equivalent to a single force, this single force intersects Δ at a point C, "the center of inertia relative to the Δ axis." It can easily be verified that Δ is then a principal axis of inertia and C is the point relative to which Δ is a principal axis. The author calls the cone of inertia relative to a point O the cone formed by all the principal axes passing through O (the cone of Staude-Ampère). The author reestablishes some known results relating to permanent rotations for a free or heavy solid having a fixed point. Ratip Berker, Turkey

617. Victor L. Nadolschi, A new integrability case in the motion of a rigid body about a fixed point (in French), Ann. Sci. Univ. Jassy, Sect. I, 30, 43–74 (1948).

The true Euler equations for the motion of a rigid body, namely $\dot{p} + (C - B)qr = L$, $\dot{q} + (A - C)rp = M$, $\dot{r} + (B - A)pq = N$, reduce in the symmetric case, $A = B$, to a single linear differential equation of the form $2f(t)\ddot{p} + f'(t)\dot{p} + 2p = g(t)$, where $f(t)$ and $g(t)$ are known functions of t if L, M, and N are known functions of t. This last differential equation reduces to the obviously integrable form $(d^2/d\tau^2 + 1)p = G(\tau)$ if the new independent variable $\tau = \int [f(t)]^{-\frac{1}{2}}dt$ is used. The paper contains many elaborations and applications of the two statements just made.

D. C. Lewis, USA

618. Cornelius Lanczos, The variational principles of mechanics, Univ. of Toronto Press, Toronto, 1949, 322 pp., 22 figs. Cloth 5.75 × 8.5 in., $5.75.

This book is devoted to the basic principles of classical mechanics. The first four chapters contain a gradual exposition of the elements of the subject, in a fashion devoid of mathematical rigor. The value of the work lies in the remaining two thirds, where in treating the Lagrangian equations, the canonical equations, canonical transformations, Hamilton's method, and Jacobi's method the author produces a synthesis, comparison, and contrast available nowhere else. There are many original details. Particularly valuable is the use of parametric forms relating the conservative and nonconservative cases, and the extraordinarily careful distinction between Hamilton's method and Jacobi's method. C. A. Truesdell, USA

619. F. Fumi, Remarks on the formulation of the Hamilton-Jacobi equations (in Italian), Atti Accad. Ligure 4, 103–118 (1948).

620. Luigi Castoldi, Appell's "abstract motions" and a new example of nonlinear anholonomic constraints (in Italian), Boll. Un. Mat. Ital. 2, 221–228 (1947).

The author describes an explicit physical example of a nonholonomic system with a constraint equation nonlinear in the velocities; this system avoids a certain kind of limiting process usually involved in setting up such examples. It is, however, of course impossible to avoid all limiting processes.

D. C. Lewis, USA

621. Luigi Castoldi, Inertia forces in Lagrangian systems and their conservative character in some particular cases (in Italian), Pont. Acad. Sci. Acta 11, 63–69 (1947).

These remarks on the so-called inertial forces in Lagrangian systems center attention on a simple condition for conservativity and (in the case of a single particle) the well-known connection with forces of Lorentz type.

D. C. Lewis, USA

622. P. Locatelli, An outline of analytical mechanics in the dynamics of structures (in Italian), Rend. Sem. Mat. Fis. Milano 18, 124–139 (1947).

623. R. Kashanin, The general equations of the motion of a system of material points of given constraints (in French, with Serbian summary), Acad. Serbe Sci. Publ. Inst. Math. 2, 116–130 (1948).

An elementary but very general derivation of the equations of motion of any dynamical system (including nonholonomic systems). D'Alembert's principle and Gauss's principle of least constraint are discussed.

D. C. Lewis, USA

624. Henri Mineur, Reduction of a quadratic form in the canonical linear group (in French), C. R. Acad. Sci. Paris 225, 1254–1256 (1947).

The author gives a canonical form for the quadratic terms in the Hamiltonian of a conservative holonomic system having multiple characteristic exponents. Actually, he mentions only linear systems, but his result is evidently applicable to any analytic sys-

tem in the neighborhood of an equilibrium point as indicated above.

D. C. Lewis, USA

625. **Émile Cotton, On certain connections between the geometry of the Riemann spaces and the classical rational mechanics** (in French), Bull. Soc. Math. France 78, 1–19 (1948).

This is old material already developed further in papers of Schouten, Vrănceanu, Synge, Horák, Wundheiler, and others.

Ed.

626. **Jean Chazy, On transformations of canonical variables** (in French), C. R. Acad. Sci. Paris 225, 1041–1044 (1947).

Several well-known theorems on contact transformations.

Ed.

627. **Anton Bilimovitch, On the canonic transformation of the equations of a nonholonomic system** (in French, with Serbian summary), Acad. Serbe Sci. Publ. Inst. Math. 2, 108–115 (1948).

A comparison between results of the author [C. R. Acad. Sci. Paris 158, 1064–1068 (1914)] and those of L. Marchetti [Ann. Scuola Norm. Super. Pisa 10, 199–208 (1941)]. Both papers treated the same problem and used the same methods with slight variations, so that it is not surprising that the results are shown to be equivalent.

D. C. Lewis, USA

628. **K. P. Persidskiĭ, On the stability of the solution of an infinite system of equations** (in Russian), Prikl. Mat. Mekh. 12, 597–612 (Sept.-Oct. 1948).

The author considers infinite systems of differential equations of the type (1) $dx_s/dt = w_s(x_1,x_2,\ldots,t)$, $s = 1,2,\ldots$, where it is assumed, inter alia, that

$$|w(x_1,x_2,\ldots,t) - w(x_1',x_2',\ldots,t)| \leq A(t)\sum_{i=1}^{\infty} a_{si}|x_i - x_i'||,$$

$s = 1,2,\ldots$, in the region defined by $t \geq 0$, $|x_i| \leq R > 0$, ${}_i\Sigma a_{si} \leq L$, and that $w_s(0,0,\ldots,t) = 0$, $s = 1,2,\ldots$. In the first part of the paper, using the method of successive approximations, the existence of a solution determined by the initial conditions $x_s(0) = c_s$ is demonstrated. It is also shown that a "méthode des réduites" is valid, namely, the solution of $dx_s/dt = w_s(x_1,x_2,\ldots,x_N,0,0,\ldots,t)$ approaches the solution of (1) as $N \to \infty$.

In the second part of the paper, the author turns to the question of the stability of the "trivial" solution, $x_s = 0, s \geq 1$. First the homogeneous equation of first approximation is discussed, $dx_s/dt = {}_i\Sigma p_{si}(t)s_i(t)$, $s = 1,2,\ldots$; then the nonhomogeneous linear equation, $dx_s/dt = {}_i\Sigma p_{si}(t)s_i(t) + f_s(t)$, and then finally, using the second method of Liapounoff, the nonlinear equation (1), all under certain hypotheses on w_s, $p_{si}(t)$ and $f_s(t)$.

R. Bellman, USA

629. **A. A. Shestakov, The behavior of the integral curves of a system of ordinary differential equations in the neighborhood of a singular point** (in Russian), Doklady Akad. Nauk SSSR 62, 171–174 and 591–594 (1948).

The equations

$$(1)\quad \dot{x}_1 = \sum_{j=m}^{\infty} c_j x_1^j,\quad \dot{x}_i = \sum_{j=1}^{n} a_{ij}x_j + X_i(x_1,x_2,\ldots,x_n),\quad i = 2,\ldots,n,$$

are considered, $O{:}x_1 = \ldots = x_n = 0$ being an isolated singular point and the X_i being power series beginning with terms of at least the second degree. The characteristic roots λ_j of the matrix (a_{ij}), $i,j = 2,\ldots,n$, are supposed to have nonvanishing real parts. Theorem 1. If $\Re(\lambda_j) > 0$, $j = 1,\ldots,k-1$; $\Re(\lambda_j) < 0$, $j = k,\ldots,n$, and if $c_m > 0$, then given any system of k sufficiently small numbers $x_1^0,\ldots,x_k^0 (x_1^0 > 0)$, there is one and only one system $x^0_{k+1},\ldots,x_n^0$ such that the solution passing through the point $(x_1^0,\ldots,x_n^0)$ tends to O as $t \to -\infty$. The singular point is classified as a node, a generalized saddle of the 1st, 2nd or 3rd type or a saddle-node, according to the signs of c_m and of the $\Re(\lambda_j)$ and whether m is even or odd. Theorem 2. If the λ_j are real and negative, $m \geq 2$, the solutions tending to O are tangent at the origin to the curve defined by equating to zero the second members of the last $n-1$ equations (1).

The second paper generalizes the preceding results to systems of the form

$$dx_1/dt = X_1(x_1),\ dx_i/dt = \varphi_i(x_1 x_i) + X_i(x_1,x_2,\ldots,x_n).$$

He assumes that $O{:}x_1 = \ldots = x_n = 0$ is an isolated singular point, that $x_1 = 0$ is an isolated root of X_1, that $J = {}_0\int^{x_1} X_1^{-1}dx_1$ diverges, that $0 < m < [\varphi_i(x_1,x_i) - \varphi_i(x_1,\bar{x}_i)]/(x_i - \bar{x}_i) < M$ if $x_i \neq \bar{x}_i$ $(i = 2,\ldots,n)$ and that the X_i have continuous first partial derivatives vanishing at the origin. Theorem 2 of the previous paper is generalized as follows: In order that the integral curves which tend to O enter the origin along one and only one direction, the assumption $dX_1/dx_1 \to 0$ as $x_1 \to 0$ is sufficient.

Courtesy of Mathematical Reviews J. L. Massera, Uruguay

630. **R. Mazet, On the stability of motions which begin without sliding** (in French), Rech. aéro. Paris no. 10, 11–23 (July-Aug. 1949).

The author discusses the problem of the effect of the coefficient of friction on the sliding movements of bodies in contact. As stated, the problem is: Given a solid S acted upon by external forces and having a certain velocity, in contact with another body S' at a single point: what will be the nature of the subsequent movement? It is shown that depending on the coefficient of friction, as related to the acceleration of the body S with respect to S' and the reaction of S' upon S, four cases may exist, i.e., the contact exists without slipping or with slipping, the solids separate, or bouncing with increasing amplitude may occur. The various cases are discussed in considerable detail and criteria are developed covering each case. The nomenclature is in some cases difficult to follow.

R. G. Wilson, USA

631. **Antonio Pignedoli, On the problem of a funicular railway: motion of a heavy rigid body whose one point is constrained to move without friction along a cable oscillating about a rectilinear equilibrium configuration** (in Italian), Atti Sem. Mat. Fis. Univ. Modena 2, 149–169 (1948).

632. **I. Tzénoff, On the deformation of an infinitely small element of a continuous medium** (in French), C. R. Acad. Bulgare Sci. Math. Nat. 1, 17–20 (1948).

The paper deals with the kinematical interpretation of the deformation tensor $(v_{ij} + v_{ji})/2$.

Ed.

633. **T. Hisada and H. Tsugawa, Studies on rolling friction** (in Japanese), J. mech. Lab., Min. Commerce & Ind. Japan 2, no. 2, 13–19 (Aug. 1948).

Rolling friction of bearing rollers on a block-gage surface is investigated experimentally by means of the oscillation method, and the following relation is obtained:

$$M_f = \text{const } (pr^3)^{1/2}$$

where M_f is the frictional moment due to rolling, p the pressure per unit length of the roller, and r the radius of the roller. By assuming that friction is caused by an adhesion force F proportional

to the contact area, the authors obtain theoretically, for the range below the elastic limits of the materials,

$$M_f = Fr = \text{const}\ (pr^3l^2/E)^{1/2}$$

where l is the length of the roller and E Young's modulus. This equation coincides with the above experimental relation. The authors discuss also the effects of hardness, roughness, oiling, etc.

Norimune Soda, Japan

634. Carl C. Saal, An evaluation of factors used to compute truck performance, Soc. Auto. Engrs. quart. Trans. 3, 215–228 (Apr. 1949).

The results of a study to determine practical values for the performance factors of trucks are presented here. Relationships for determining rolling resistance, air resistance, grade resistance, chassis frictional resistance, power loss due to altitude, and the mass-equivalent constant are given that closely approximate actual performance figures. It is pointed out, however, that there exists considerable controversy in the industry over some of these factors, especially those for air resistance and chassis frictional resistance, indicating that an experimental research program is definitely required to develop sound engineering values for these two resistances.

Author's summary

Gyroscopics, Governors, Servos

(**See also Revs. 617, 701**)

635. L. I. Tkachev, On the 84-minute period for a system with coupled or free gyroscopes (in Russian), Prikl. Mat. Mekh. 13, 217–218 (1949).

The formula $T = 2\pi(R/g)^{1/2}$ derived by M. Schuler [Phys. Z. 24, 344–350 (1923)] gives a period of 84 minutes for a certain oscillation of a pendulum or gyroscope (R is the radius of the earth). This author shows that the same formula also applies to the period of a disturbance of one gyroscope of a coupled system.

Courtesy of Mathematical Reviews R. E. Gaskell, USA

Vibrations, Balancing

(**See also Revs. 605, 632, 659, 676, 702, 708, 736, 792, 802**)

636. M. Ya. Leonov, The parametric representation of quasiharmonic oscillations (in Russian), Doklady Akad. Nauk SSSR 62, 161–162 (1948).

Solutions of the equation (1) $\ddot{x} + 2\gamma(\theta)\dot{x} + x = 0$ ($\dot{x} = dx/d\theta$) are given in the form $x = \sin\varphi \exp(-2\,{}_c\!\int^{\theta} \cos^2\varphi d\theta)$, where φ is the general solution of (2) $d\varphi/d\theta = 1 + \gamma \sin 2\varphi$, and c is a constant. The author introduces the function $\mu(\varphi) = \gamma d\theta/d\varphi$. The case when γ is periodic of period L, ${}_0\!\int^L \gamma d\theta > 0$, is considered. The author states that, for the existence of unstable solutions of (1): (a) it is necessary that a solution φ_p of (2) exist with the property $\varphi_p(\theta + L) = \varphi_p(\theta) + n\pi$, where n is an integer; (b) if (a) is satisfied, it is necessary and sufficient that

$$\left|{}_0\!\int^{n\pi} \mu \cos 2\varphi d\varphi\right| > {}_0\!\int^{n\pi} \mu d\varphi.$$

No proof is given of the first statement. The second one is apparently incorrect.

Courtesy of Mathematical Reviews J. L. Massera, Uruguay

637. B. P. Demidovich, Periodic solutions of nonlinear systems of the second order of ordinary differential equations whose second members are periodic relative to the independent variable (in Russian), Doklady Akad. Nauk SSSR 61, 601–603 (1948).

The author considers systems of the form

$$\dot{x} = f_{10} + xf_{11} + yf_{12},\ \dot{y} = f_{20} + xf_{21} + yf_{22}, \qquad (1)$$

where the f are real continuous functions of x, y, t, periodic in t of period T, such that $f_{i0} = o(r)$ uniformly when $r^2 = x^2 + y^2 \to \infty$ and such that the solution of (1) through any (x_0, y_0, t_0) is unique and exists for $-\infty < t < +\infty$. Let (1′) be the "abbreviated" system obtained from (1) by dropping the terms f_{i0}. Lemma. Let $U = Ax^2 + 2Bxy + Cy^2$ be a positive definite form with constant coefficients whose derivative $\dot{U} = dU/dt$ (by virtue of (1′)) satisfies $\inf \dot{U}/r^2 > 0$ for $r \geq r_0$. Then (1) has at least one periodic solution of period T.

Two criteria for the existence of periodic solutions of period T (essentially in terms of the characteristic roots of the matrix f_{ij}) are derived from this lemma. The reviewer observes that the lemma, which the author deduces from Schauder's fixed point theorem, can be easily obtained from a theorem by Levinson [Ann. of Math. 45, 723–737 (1944)]. No proofs are given.

Courtesy of Mathematical Reviews J. L. Massera, Uruguay

638. F. de Kok, On the determination of small oscillations with two degrees of freedom (in Dutch), Simon Stevin 25, 228–230 (1947).

The author determines the vibrations about equilibrium of a system with two degrees of freedom without using the theory of quadratic forms.

Courtesy of Mathematical Reviews J. Haantjes, Holland

639. F. M. Dimentberg, On transverse vibrations of a rod with distributed mass in the presence of damping (in Russian), Prikl. Mat. Mekh. 13, 51–54 (1949).

The author derives a general expression for the complex natural frequencies, $\Omega_k = \omega_k + i\alpha_k$, of transverse vibrations of uniform beams with both internal damping (proportional to time rate of bending deformation) and external damping (proportional to displacement velocity). In Ω_k, ω_k is the real oscillation frequency and α_k the amplitude decremental coefficient. It is brought out that internal damping acts to set an upper bound to the real spectrum of natural frequencies despite the infinite number of degrees of freedom possessed by a beam. In the latter portion of the paper Biot's concept of "dynamic stiffness" is extended to include a pinned-free bar with a forcing moment of the form $M = M(0) \exp(i\Omega t)$, acting at the pinned end. The dynamic stiffness then appears in the form

$$K = -I\Omega^2\Pi(1 - \Omega/\Omega_k)(1 + \Omega/\Omega'_k)/\Pi(1 - \Omega/{}^*\Omega_k)(1 + \Omega/{}^*\Omega'_k)$$

where Ω_k, Ω'_k are the complex conjugate resonance frequencies, ${}^*\Omega_k$, ${}^*\Omega'_k$ are the complex conjugate antiresonance frequencies, and I the mass moment of inertia of the bar about its pinned end.

Walter W. Soroka, USA

640. D. Williams, Rapid estimates of the higher natural frequencies and modes of beams and hence of the stresses induced by transient and periodic loads, Reissner Anniv. Vol., J. W. Edwards, Ann Arbor, 152–162 (1949).

The method is based on the assumption that loops in the elastic line of a vibrating beam at any of its higher natural frequencies can be approximated by sinusoidal half waves. Then the ratio of two consecutive frequencies will be $p_{n+1}/p_n = [(N_n + 1)/N_n]^2$, where N_n is the number of "effective" loops at frequency p_n. The number N_n, that is generally not an integer, can be readily found for the fundamental mode ($n = 1$), and from this all the higher frequencies and modes can be deduced with remarkable accuracy. The only exception appears to be the cantilever beam, which forms no loop in the first mode, and in this case data from the second mode should be used in deducing higher frequencies.

The method works equally well for torsional vibrations, where the frequencies vary linearly with the number of effective half waves, and for beams of variable cross section, where frequencies of higher order can be calculated from the ratio of the first two natural frequencies.

The author shows that the frequency value and the number of loops in any of the higher modes also define the corresponding lengths and amplitudes of the loops. For built-up beams of varying web height $2h$, where most of the weight is concentrated in the flanges of equal cross-sectional areas A, it is shown that the lengths of the loops vary directly with $h^{1/2}$ and their amplitudes inversely with $Ah^{1/2}$, independently of the order of the mode of vibration.

The last section of the paper deals with the calculation of stresses produced by impulsive or transient loads whose time-history is known. The proposed procedure consists of determining the effect of the transient load for each mode, by equating the corresponding normal component of the applied force to the sum of the resisting elastic and inertia forces in the beam, and deriving the final result by summation of the component effects.

M. Hetényi, USA

641. Antonio Pignedoli, Vibrations of a circular plate under radial pulsating pressure (in Italian), Atti. Sem. Mat. Fis. Univ. Modena 2, 3–19 (1948).

642. Giuseppe Colombo, On small oscillations of a heavy conical surface (in Italian), Ist. Veneto Sci. Lett. Arti. Sci. Mat. Nat. 106: 172–179, 180–183 (1948).

The author derives an intrinsic differential equation of motion for the small oscillations, about a configuration of equilibrium, of a heavy inextensible membrane which is bounded by two fixed straight line segments OA and OC, and by a variable arc ABC. It is assumed that the oscillations are such that the membrane is always a portion of a (variable) conical surface.

In the second note the author discusses the reality and signs of the eigenvalues of the problem, and also certain questions relating to the calculation of solutions of the differential equation.

Courtesy of Mathematical Reviews L. A. MacColl, USA

643. Giuseppe Colombo, On the integral equation with nucleus dependent on the parameter, for normal vibrations of a sphere immersed in a fluid (in Italian), R. C. Semin. Mat. Padova 17, 29–38 (1948).

Starting from the differential equation and boundary conditions discussed by Laura [R. C. Accad. Lincei, ser. 5, vol. 21] for the radial vibrations of a sphere immersed in a liquid, and transforming it into an integral equation, there results a symmetric nucleus which is a rational function of a parameter and is positive definite. In general, the characteristic values are found to have a negative real part. In the special cases of zero or infinite density of the medium, pure vibrations, without damping, result.

Edward Saibel, USA

644. Jules Haag, On vibrations of certain spring-supported machines (in French) J. Math. Pures Appl. (9) 25, 257–288 (1947).

The author considers forced vibrations of a general mechanical system which can be described in the following way. A massive table is supported by springs, and is itself the support of one or more subsystems. Each subsystem contains a body which can rotate about an axis (fixed with respect to the table) or can vibrate in two dimensions about a position of equilibrium. The subsystems are maintained in periodic motion by a motor, which is itself regarded as one of the subsystems. The paper deals chiefly with the resonance frequencies of the system and with the amplitudes of motion of the several parts. In particular, the author is concerned with the problem of designing the system so as to avoid the occurrence of resonance and the associated large motions. After a derivation of the differential equations of motion for the general case, there follows an extensive discussion of several relatively simple special cases which are of practical importance. This discussion includes numerical calculations which illustrate various parts of the theory.

Courtesy of Mathematical Reviews L. A. MacColl, USA

Wave Motion, Impact

(See also Revs. 681, 692, 713, 737, 738)

645. Z. Horák, Theoretical formula for restitution coefficients of imperfectly elastic bodies (in French), Bull. Éc. polyt. Jassy 3, 218–225 (Jan.–June 1948).

The author proposes the formula

$$k_{12} = \frac{(1 - \sigma_1^2)k_{11}E_2 + (1 - \sigma_2^2)k_{22}E_1}{E_2(1 - \sigma_1^2) + E_1(1 - \sigma_2^2)}$$

for the coefficient of restitution in the collision of two bodies with individual coefficients k_{11}, k_{12} and elastic moduli E_1, σ_1, E_2, and σ_2. Hodgkinson's formula is derivable from this one by setting $\sigma_1 = \sigma_2 = 0$. The hypothesis is used that the initial deformation satisfies Hertz's relation for contact pressure, and that the ratio of the deformations remains constant during impact. Application is made to collision of glass and ivory spheres on plates.

D. L. Holl, USA

646. N. V. Zvolinskiĭ, Asymptotic investigation of the solution of the problem of propagation of a point disturbance in an elastic semispace covered by a layer of compressible fluid (in Russian), Doklady Akad. Nauk SSSR 65, 145–148 (Mar. 1949).

In a previous paper [same source, 59 (1948); Rev. 1, **943**] the author described the title process by means of certain three functions. However, the representations there obtained are not convenient for the study of the process when $t \to \infty$. The present paper transforms the representations appropriately, and studies the asymptotic behavior of the process. Ed.

Elasticity Theory

(See also Revs. 642, 645, 646, 670, 676, 691, 793)

647. Mahmut Tanrikulu, Ordinary zero places in a body under plane stress (in English, with Turkish summary), Rev. Fac. Sci. Univ. Istanbul (A) 13, 205–235 (1948).

This paper purports to deal with plane stress, in which the stress vanishes across all planes parallel to a fixed plane. However, in plane stress there are other equations of compatibility in addition to the one given by the author, and hence the surviving stress components are considerably restricted. The equations of equilibrium and compatibility which the author uses actually correspond to plane strain or to generalized plane stress. The paper is concerned with the distribution of stress lines, i.e., curves tangent at each point to a principal direction of stress. The differential equation of the stress lines is $y'/(1 - y'^2) = P(x, y)/Q(x, y)$, where P and Q are simple functions of the stress components. If P and Q are both homogeneous polynomials of degree n, the origin is called an ordinary zero place of order n. The paper gives an exhaustive discussion of the case $n = 1$.

J. L. Synge, Ireland

648. Bibhutibhusan Sen, Direct determination of stresses from the stress equations in some two-dimensional problems of elasticity. V. Problems of curvilinear boundaries, Phil. Mag. (7) 39, 992–1001 (1948).

[For part IV, see same source 36, 66–72 (1945).] It is shown that the stress components for a state of generalized plane stress can be expressed in terms of $\Theta = \widehat{xx} + \widehat{yy}$, which is harmonic, and explicitly in terms of a pair of conjugate harmonic functions φ and ψ. This formulation facilitates conversion into the orthogonal curvilinear system of coordinates $\xi + i\eta = f(x + iy)$. Appropriate choices of Θ, φ and ψ are demonstrated to determine the stress distributions for a plate under uniform tensile stress at infinity, and containing a hole in the form of (1) the inverse of an ellipse, (2) the loop of a lemniscate, (3) an elliptic limaçon, and (4) an approximate square. In addition the problem of an elliptic hole in an infinite plate with uniform normal pressure on the elliptic boundary is solved by this method. E. H. Lee, USA

649. Mauro Picone, Existence and calculation of the solution of a certain boundary-value problem for a system of equations of two-dimensional elasticity (in Italian), Boll. Un. Mat. Ital. 3, 4–6 (1948).

This note states the simplifications resulting when the author's general method of integrating the equations of elasticity [see Rev. 1, **596**] is applied to a plane system. C. A. Truesdell, USA

650. M. M. Filonenko-Borodich, The bending of a rectangular plate with two clamped opposite edges (in Russian, with English summary), Vestnik Moskov. Univ. 1947, no. 3, 29–36.

This is a sequel to the author's paper [Prikl. Mat. Mekh. 10, 193–208 (1946)] in which the system of "almost orthogonal functions" $P_n(x) = \cos n\pi x/a - \cos (n+2)\pi x/a$, $n = 0, 1, 2, \ldots$, is applied to solve approximately the problem of deflection of a thin elastic rectangular plate clamped on the edges $x = 0$, $x = a$, and with arbitrarily prescribed boundary conditions along $y = 0$ and $y = b$. The normal load $g(x, y)$ is symmetric with respect to the line $x = a/2$. The approximate deflection w is sought in the form $w = {}_m\sum_0^n P_{2m} f_{2m}(y)$, where the functions $f_{2m}(y)$ satisfy the system of fourth-order differential equations arising from the deflection equation $\nabla^4 w = g(x, y)/D$. The solution for rectangular plate clamped along all edges is indicated. I. S. Sokolnikoff, USA

651. Giuseppe Aymerich, Conjugate force configurations in plane elasticity (in Italian), Rend. Sem. Fac. Sci. Univ. Cagliari 16, 145–148 (1948).

652. R. S. Rivlin, A note on the torsion of an incompressible highly elastic cylinder, Proc. Cambridge phil. Soc. 45, 485–487 (July 1949).

The author gave earlier [sect. 14 of Phil. Trans. R. Soc. London (A) 241, 379–397 (1948)] exact finite formulas for the stresses in the torsion of a right circular cylinder of an isotropic incompressible perfectly elastic material, the result being valid for any form of strain energy and for strain of any magnitude. In the present note the author obtains a simpler form for this important result. C. A. Truesdell, USA

653. Domenico Gentiloni-Silverj, Integration of the equation $\Delta^2\chi = 0$ in a three-dimensional elasticity problem (in Italian), Consiglio Naz. Ricerche. Pubbl. Ist. Appl. Calcolo no. 206, 44 pp. (1947).

The problem discussed is that of forcing a cap onto a rigid disk of diameter slightly greater than the internal diameter of the rim of the cap. Mathematically, this means solving a three-dimensional elastostatic problem, the displacement being given on the inside of the rim of the cap, and the stress across the rest of the surface of the cap being zero. However, on account of the symmetry of the problem, the assignment of displacement is equivalent to assignment of a component of strain, and so (through the stress-strain relations) equivalent to a condition on the stress components. Thus the surface conditions are expressible entirely in terms of stress. The author uses the known fact that stresses expressed in terms of any axially symmetric three-dimensional biharmonic function χ satisfy the equations of equilibrium and compatibility. The purpose of the paper is to choose a biharmonic χ which makes the errors in the boundary conditions small. To this end there is taken as nth approximation $\chi = {}_i\Sigma_1^n C_i\chi_i$ where χ_i is a biharmonic polynomial of degree i in cylindrical coordinates ρ, z, and C_i are any constants. The error is computed by integrating over the surface of the cap the sum of the squares of the differences between the required stresses and those given by χ, the constants C_i being chosen to minimize the integral. At this stage arithmetical values are introduced and computations made up to $n = 5$. [In the true title of the paper, the equation reads $\Delta^4\chi = 0$ rather than $\Delta^2\chi = 0$.] J. L. Synge, Ireland

654. M. Picone, On the calculation of the deformation of a built-in elastic solid (in French), Proc. Seventh int. Congr. appl. Mech. 1, 1–9 (Sept. 1948).

A definition of a built-in support for an elastic member C is given which allows for the elasticity of the support. The support C' is taken to be a homogeneous elastic solid (whose elastic constants differ from those of C) extending to infinity in all directions with a single internal boundary-surface, part of which is in contact with C. Displacements and stresses are taken to be continuous at the intersurface, and the displacements in C' vanish at infinite distance.

A formal solution for the stress and strain distribution resulting from a given body force and surface-stress distribution is outlined in terms of an infinite family of linearly independent bihyperharmonic functions. Stephen H. Crandall, England

655. W. Schmeidler, Reduction of thermal stresses in an elastic body to a buckling-bending problem (in German), Z. angew. Math. Mech. 28, 92–94 (Mar. 1948).

A homogeneous isotropic elastic body is in equilibrium in a known steady-temperature field ϑ; there is no body force and the surface is stress-free. It is proved that the elastic displacement is expressible as the sum of (1), a displacement D_1 given explicitly as a volume integral whose value depends on ϑ, and (2), the displacement that would be produced by certain fictitious stresses whose values depend upon D_1 and ϑ, when the temperature is taken as zero. A more appropriate title would be: "The determination of the thermal stresses in an elastic body."

H. G. Hopkins, England

656. W. Schmeidler, Thermal stresses in a body (in German), Z. angew. Math. Mech. 28, 54–59 (Feb. 1949).

A homogeneous isotropic elastic body is in equilibrium in a known steady-temperature field; there is no body force and the surface is stress-free. Let D be the elastic vector displacement. It is proved that div D and curl D satisfy certain linear integral equations whose kernels involve a Green's function of the second kind for the region occupied by the body. Since it is possible to write $D = \text{grad}\,\phi + \text{curl}\,H$, where ϕ and H are determined by certain integral formulas when div D and curl D are known, the

determination of D is thus reduced to the solution of these integral equations. The integral equations are not independent and involve both surface and volume integrals.

H. G. Hopkins, England

657. **Carlo Cattaneo, A theory of second-order elastic contact** (in Italian), Univ. Roma Rend. Mat. Appl. 6, 505–512 (1947); see Rev. 1, **787**.

658. **Carlo Cattaneo, A theory of second-order elastic contact: oblique compression** (in Italian), Rend. Sem. Fac. Sci. Univ. Cagliari 17, 13–28 (1948).

659. **V. G. Gogoladze, Reflection and refraction of elastic waves** (in Russian, with English summary), Acad. Sci. URSS Publ. Inst. Seismolog. no. 125, 43 pp. (1947).

The author studies the reflection and refraction of elastic waves, as well as the seismologically important problem of the Rayleigh surface waves. Chapter 1 discusses plane waves: the reflection and refraction of longitudinal waves, energy distribution of heterogeneous and surface waves, and Rayleigh's equation. A deeper study of the latter is taken up in chapter 2, especially of the roots of the Rayleigh function on the Riemann surface, at various hypotheses concerning the velocity. A criterion for the existence of Rayleigh's surface waves is also given. The paper concludes with a bibliography.

Courtesy of Mathematical Reviews V. A. Kostitzin, France

Experimental Stress Analysis

(**See also Revs. 665, 711**)

660. **William J. Eney, A large displacement deformeter apparatus for stress analysis with elastic models,** Proc. Soc. exp. Stress Anal. 4, no. 2, 84–93 (1949).

The author's apparatus is an adaptation of the well-known Beggs deformeter scheme, with the principal difference that it provides for much larger deformations, which can be measured by ordinary ruled scales. It is claimed that the nonlinear effect of the large deformations can be eliminated for all practical purposes by taking the difference between deflection ordinates produced in opposing directions from the neutral, unstressed, configuration of the model.

M. Hetényi, USA

661. **William J. Eney, Studies of continuous bridge trusses with models,** Proc. Soc. exp. Stress Anal. 4, no. 2, 94–105 (1949).

Models were prepared of continuous bridge trusses in scales 1:70 and 1:100, for the main purpose of experimentally studying influence lines for pier reactions, truss deflections and jacking forces in the various stages of erection of the structures. The extensibility of each bar in the model was provided by means of brass-leaf springs, capable of carrying compressive as well as tensile loads. The model was made elastically similar to the prototype by maintaining for all bars the same ratio between the spring constants in the model and the corresponding spring constants in the actual structure. The preparation and calibration of bolted brass springs for the model are discussed in detail, and the illustrations include numerous influence lines derived by the method.

M. Hetényi, USA

662. **R. King, The investigation of internal stresses by physical methods other than X-ray methods,** Inst. Metals Monogr. Rep. Ser. no. 5, 13–23 (1948).

A survey is given of various physical methods, such as those based upon magnetostriction, electrical resistivity and internal friction, which are used to detect the internal stresses in cold-worked and otherwise treated materials. In general, the results obtainable are of a qualitative nature only.

J. A. Haringx, Holland

Rods, Beams, Shafts, Springs, Cables, etc.

(**See also Revs. 604, 640, 652, 681, 683, 684**)

663. **Florin Vasilesco, Buckling of beams of uniform cross section and of variable moment of inertia** (in French), Ann. Sci. École Norm. Sup. 64, 247–274 (1948).

Beams of uniform sections but variable moment of inertia are loaded by end thrust and (in some cases) lateral forces and end moments. The conventional questions concerning the buckling of such beams are asked. The attack is less efficient and has no more accuracy than the conventional Fredholm equation approach.

G. F. Carrier, USA

664. **W. M. Stone, Note on a paper by N. J. Durant,** Philos. Mag. 39, 988–991 (1948).

The author gives an application of the generalized Laplace transformation to the problem of the continuous beam of N equal spans. Cf. Samuelson, Bull. Amer. Math. Soc. 52, p. 240 (1946).

A. E. Heins, USA

665. **C. A. Sciammarella and M. A. Palacio, Photoelastic analysis of a beam of great height** (in Spanish), Cienc. Tecn. 113, no. 569, 249–275 (Nov. 1949).

This paper deals with an experimental test of a theoretical work already reviewed here [Revs. 1, **57**, **1608**; 2, **172**]. The authors used a photoelastic interferometer and a "perspex" model of a beam of height equal to span, uniformly loaded on the upper boundary. They determined both principal stresses on the vertical axis of symmetry and they compared the values obtained with those given by the theoretical methods mentioned above. The maximum tensile stress obtained experimentally is more than 15% greater than the maximum tensile stress obtained using the finite-difference method; and about 10% smaller than the stress given by the Ritz method. The authors determined also the stress for all points on the boundary, and found that the maximum tensile stress on the lower boundary is not at the axis of symmetry.

The theoretical solutions the authors compared with their experimental results correspond to beams loaded on the lower boundary, and although they state that these different boundary conditions do not influence the normal longitudinal stresses, the reader would expect to see this proved.

The authors found the photoelastic interferometer (Favre type) very easy to use. They are not, however, aware that in this country this instrument is used regularly at the Bureau of Reclamation. However, it does not seem to the reviewer that it has been proved that this instrument is more practical than the conventional polariscope, especially when the sum of the principal stresses can be determined extremely fast by the iteration method in specimens of rectangular shape for which all boundary conditions are known.

A. J. Durelli, USA

666. **E. Foulon, A study of possible lightening of rolled-beam sections** (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale 3, 199–314 (1948).

The author considers I-beams with profile consisting of a rectangular web and equal rectangular flanges, and thus specified by four dimensions. He raises the question: If one holds fixed the section modulus (maximum moment of inertia/depth of

beam) and the ratio of web-breadth to thickness, is it possible to choose dimensions which render the area, and hence the weight, a true minimum? If no further restrictions are made, there are two independent variables, and minimization can be attempted with respect to these. An apparent solution is obtained, but further study shows that the area determined is stationary, but neither minimum nor maximum. However, if one additional condition is imposed the author finds that true minima exist. He considers three cases in which one takes in turn the following quantities to be prescribed: (1) flange breadth; (2) beam depth; (3) ratio of flange breadth to beam depth. For each of these cases the author determines the proportions for minimum weight, and plots curves and diagrams giving various dimensionless quantities as functions of the parameters.

Comparisons are given between the weights of the various "ideal sections" and those of standard Belgian, American, British and German profiles. Considerable economics are said to be obtainable by the ideal sections, which give for example a saving in weight of 26% over a typical Belgian section. The author discusses the required flange breadth—thickness ratio from the point of view of buckling—and also discusses the lateral stability of the ideal sections. Finally, he proposes a series of "economic profiles" proportioned so as to obtain maximum economy consistent with requirements of manufacturing and assembling.

P. S. Symonds, USA

667. S. E. Beerman, Solutions in polynomials for thin-wall beams (in Russian), Doklady Akad. Nauk SSSR 65, 283–286 (Mar. 21, 1949).

The author presents solutions of three specific problems relating to thin-wall beams. The methods of solution are based on previously derived equations using functions of a complex variable [same source, 62, 187–190 and 305–308; see Revs. 2, **165** and **168**].

The problems treated in this article are:

(1) Stresses and strain in a thin-wall, cantilever, rectangular box beam, loaded along all four corners by antisymmetric linearly increasing vertical forces. This corresponds to a torque linearly increasing from zero at the free end to a maximum at the fixed end.

(2) Stresses and strains in a thin-wall, cantilever, rectangular box beam loaded by two couples consisting of forces in the plane of the boundary walls. This solution corresponds to the case of a torque applied to the beam by means of a rigid diaphragm.

(3) Location of center of twist of a thin wall, cantilever beam having a channel cross section.

The nomenclature and the derivation are indicated only by references to the previous articles.

Boris Bresler, USA

668. Odone Belluzzi, The stability of thin-wall tubes under uniform axial tension (in Italian), Mem. Accad. Sci. Ist. Bologna Cl. Sci. Fis. 3, 33–36 (1947).

669. J. M. Klichïev, On the torsion of a rectangular tube (in Russian), Acad. Serbe Sci. Publ. Inst. Math. 1, 58–61 (1947).

See Rev. 1, **229**.

670. J. Mandel, Determination of the center of twist by means of the reciprocity theorem (in French), Ann. Ponts Chauss. 118, 271–290 (May–June 1948).

The author finds formulas for locating what is usually termed the shear center or center of flexure, defined as the point through which a transverse force applied to the end of a cantilever must pass in order that the bent shape of the beam shall be one in which the mean axial component of rotation is zero. The following formulas are given for the coordinates ξ, η of this point referred to centroidal principal axes x,y:

$$I_x\xi = -\iint_\Sigma y\varphi(x,y)d\omega + \sigma/(1+\sigma)\iint_S x\psi_1(x,y)d\omega$$

$$I_y\eta = \iint_\Sigma x\psi(x,y)d\omega + \sigma/(1+\sigma)\iint_S y\psi_1(x,y)d\omega.$$

Here I_x, I_y are moments of inertia, $d\omega$ is an element of area, Σ is the area of the section, S is the area enclosed by the outer boundary, σ is Poisson's ratio, and $\varphi(x,y)$ and $\psi_1(x,y)$ are solutions of the Saint Venant torsion problem of the section, ϕ being the "warping function" and ψ_1 the "stress function."

The author is led to these formulas by a direct application of the Maxwell-Betti reciprocity theorem to the section of a cantilever beam in which the Saint Venant flexure and torsion theories are rigorously correct. He also rederives them by transforming the corresponding formulas given by the "direct method," taking the solution of the flexure problem in terms of two harmonic functions with specified normal derivatives on the boundaries. Special formulas for thin-walled sections are given. The author locates the shear center for a number of examples. In particular he shows that his formulas may be considerably more convenient than those of the direct method in the case of thin-walled closed sections, but not in the case of open sections.

The author does not refer to the work of Trefftz [Z. angew. Math. Mech. 15, p. 220 (1935)], whose formulas agree only with the approximate formulas of the present paper obtained by assuming $\sigma = 0$, i.e., by neglecting distortions in cross-sectional planes. The error caused by this assumption in the case of the thin-walled closed section taken as an example in the paper (roughly simulating an airplane wing section) is about 12%.

P. S. Symonds, USA

Plates, Disks, Shells, Membranes

(See also Revs. 604, 648, 650, 667, 678, 680, 685, 691, 708)

671. B. R. Seth, Bending of rectilinear plates, Bull. Calcutta Math. Soc. 40, 36–40 (1948).

The author relates the deflection of a simply supported plate to the torsion problem for the same boundary, and proceeds to obtain that deflection in what is essentially the conventional product-series solution.

G. F. Carrier, USA

672. Sten G. A. Bergman, Behaviour of buckled rectangular plates under the action of shearing forces, Kungl. Tekniska Hogskola, Stockholm, 1948, 166 pp. Paper, 6.5 × 9.5 in.

The principal part of this paper is devoted to a theoretical analysis of the behavior of buckled rectangular plates subjected to shearing forces along all edges with the purpose of providing a rational basis for developing design rules for shear-loaded webs of plate girders. Since the linear theory of plates does not suffice to predict the actual load-carrying capacity of plates subjected to edge loads, the author has based his analysis on von Kármán's differential equations for plates with large deflections. The nonlinear differential equations are solved approximately by using Ritz's variational method.

Complete theoretical solutions are given for a number of special cases, namely, initially plane square plates both with rigid supports and elastic supports, square plates with slight initial curvature and rigid supports, and infinitely long plates which are initially plane. The edges are assumed to be simply supported in all cases. The determination of the load which causes yielding in the buckled plate is based on the assumption that the plate material obeys Hooke's law right up to the yield

point, and further that the yield condition is defined by the hypothesis of maximum strain energy of distortion.

The theoretical results are compared with test results given by various investigators and it is found that the theory gives a fairly correct idea of the behavior of actual buckled plates. Based on the results of the theoretical analysis, a design chart is constructed from which the actual load carrying capacity of shear webs in plate girders may be determined. Examples are given which indicate that current design specifications are somewhat conservative.

This paper, which contains 166 pages, covers the subject clearly and thoroughly. It should be of interest to many engineers. Dana Young, USA

673. J. M. Klichĭev, On the stability of the deck plates of steel ships (in Russian, with Serbian summary), Acad. Serbe Sci. Publ. Inst. Math. 2, 53–78 (1948).

674. Miodrag Milosvatjevitch, Stability of rectangular plates with stiffeners under bending shear (in French), Acad. Serbe Sci. Publ. Inst. Math. 1, 121–135 (1947).

A simple supported elastic rectangular plate with elastic stiffness lying parallel to both the transverse and longitudinal edges is subjected to loads which might produce buckling. The critical magnitude of this load is determined using a Fourier series technique. G. F. Carrier, USA

675. A. Kleinlogel and E. Schmitt, Geometric method to determine the stresses in membranes of reinforced concrete (in German), Bauplan. Bautech. 3, 51–58 (February 1949).

This is an exposition of a graphical method employed by Laponche, a French engineer, to determine the membrane stresses in cylindrical shells when bending stiffness is neglected. An example is given comparing the graphical method with known analytical methods. D. L. Holl, USA

676. A. L. Gol'denveĭzer, and A. I. Lur'e, On the mathematical theory of the equilibrium of elastic shells (Survey of the work published in the USSR) (in Russian), Prikl. Mat. Mekh. 11, 565–592 (1947).

This is a condensed survey of the research literature on the subject published in Russia during the past decade. Three distinct directions are discernible: (a) theoretical investigations based on the fundamental equations of the mathematical theory of elasticity, (b) work on stability and vibrations, (c) papers concerned with the engineering applications of the theory. This survey is concerned only with the first aspect. The development surveyed in this article falls into three categories: (a) formulation of the basic equations of the theory of thin shells, which extends the classical theory of Love with the aid of modern tools of differential geometry; (b) specialization of the general three-dimensional problem of the theory of elasticity to a two-dimensional one by introducing certain geometrical hypotheses and physical assumptions; papers in this category are concerned with the analysis of the nature of the simplifications and with the study of the magnitude of errors inherent in them; (c) integration of the equations formulated in category (a). This is accomplished by replacing the complete system of equations by special systems yielding the information about the "edge effect" and the behavior of membrane or momentless shells.

In addition to the account of the general investigations falling in these categories, the survey contains a résumé of several problems of integration of systems of equations associated with specific geometrical forms. These include spherical shells, conical shells and shells with vanishing Gaussian curvature. The survey concludes with a bibliography of 48 items. I. S. Sokolnikoff, USA

677. Odone Belluzzi, A simplified computation of doubly-curved plates (in Italian), Mem. Accad. Sci. Ist. Bologna. Cl. Sci. Fis. (10) 3, 37–46 (1947).

Buckling Problems

(**See also Revs. 604, 655, 666, 668, 672, 673, 674, 676, 684, 708**)

678. P. P. Bijlaard, On the torsional and flexural stability of thin-walled open sections, Nederl. Akad. Wetensch. Proc. 51, 314–321 (1948). Review delayed.

679. E. W. Kuenzi and C. B. Norris, Longitudinally stiffened thin-walled plywood cylinders in axial compression, For. Prod. Lab. Rep. no. 1562, 30 pp. (Apr. 1948).

Data are reported for axial-load compression tests of 1050 thin-walled plywood cylinders having longitudinal stiffeners. Specimens had wall thickness of 0.05 in. with stiffeners of veneer, plywood or solid wood ranging from $^1/_{48}$ in. to 1 in. thick and spaced from $^1/_4$ in. to 6 in. around the circumference. Buckling occurred at the critical stress for the unstiffened cylinder except for some increase with closely spaced rigid stiffeners. In general the stiffened cylinders were no stronger than unstiffened cylinders having thicker walls but the same total weight.

Henry A. Lepper, Jr., USA

680. F. Reutter, On the stability of sandwich plates and bars whose center layer is of light weight with a modulus of elasticity variable in the normal direction (in German), Z. angew. Math. Mech. 28: 1–12, 132–142 (Jan., May, 1948).

The paper deals with the elastic buckling load, both for over-all instability and for wrinkling, of sandwich struts and panels, when the modulus of elasticity of the core drops from a maximum E_b at the faces to a minimum E_c in the center of the core thickness. In most of the paper an exponential variation is assumed. Throughout, Poisson's ratio is assumed to be zero in the core.

The first two sections summarize the results of a paper by the same author on the deformations and elastic restraint coefficients of the core alone, when loaded by a sinusoidal transverse loading along the boundaries [Ingen.-Arch. 16, nos. 5–6, 307–320 (1948); Rev. 3, **37**]. Infinite and finite core thicknesses are considered.

Sections 3 through 8 give the rigorous solutions for the instability of a strut on an infinite elastic foundation, for the wrinkling of the sandwich strut and for the over-all buckling of the sandwich strut, including numerical results. For the second case it is shown that antisymmetric wrinkling always corresponds to a lower load than symmetric wrinkling. The solution for the third case proves to be unsuitable for practical applications. Therefore, in section 6 an approximate energy solution is derived; the resulting formula can be considered an obvious generalization of the formula holding for a homogeneous isotropic core.

The paper concludes with a section on the optimum design of sandwich struts having inhomogeneous cores, and some brief remarks on the stability of sandwich panels of the same composition.

Since the numerical results for the wrinkling loads of sandwich struts in figs. 9 to 11 are difficult to interpret, the reviewer has checked the conclusion given in Rep. Memo. aero. Res. Counc. London no. 2143, for one particular case. This conclusion was that if the wrinkling load is compared with that of a sandwich having a homogeneous isotropic core with a modulus of elasticity E_{tr} equal to the average modulus measured under compression of

the core perpendicular to the faces, the latter load will be smaller than the actual buckling load. For the case considered, in which $E_b = 2.72\ E_c$ and $E_b = 1.72\ E_{tr}$, this conclusion was confirmed and the difference (13%) was of the same order as that (16%) in the cited Rep. Memo. F. J. Plantema, Holland

Joints and Joining Methods

(See also Revs. 681, 684, 686)

681. W. J. Krefeld, E. C. Ingalls, G. G. Luther, W. E. Ellis, C. E. Hartbower, and M. G. Salvadori, An investigation of beams with butt-welded splices under impact, Weld. Res. Suppl. 12, 373–432 (July 1947). Supplement to Rev. 3, **461**.

This paper deals with static and impact tests on plain and welded-steel beams at room and subzero temperatures, and is supplemented by a mathematical treatment of some of the fundamental aspects of the impact phenomena. The elastic strains are measured with SR-4 gages, the plastic strains with Whittemore gages, and the deflections with a phototube connected to an oscilloscope. All the techniques involved in the testing and measuring, as well as the welding techniques are described in great detail. The authors found that the impact on the beam is followed by a series of subimpacts all of which occur during the half-vibration cycle. They also compare the deflections and strains for both plain and welded beams, under successive increments of impact. All of the welded beams show a considerably lower dynamic proportional limit than the plain beams. It is strange however that under dynamic loading the deflections of the welded beams appear to be smaller than those of the plain beams. The results of static tests show a lower elastic limit in the welded beams and seem to indicate that holes in the web do not change the amount of stress on the bottom of the flange directly opposite the hole, nor do they influence the general behavior of the beams.

The results of tests conducted at low temperature (−40 F) show that the dynamic elastic modulus does not change appreciably, but that the proportional limit is higher. The authors feel however that the number of tests at low temperature was too small to reach definite conclusions.

To correlate transition temperatures obtained from small specimens and from prototypes, the authors tested large beams, and Charpy and high-constraint nick-bend specimens. These results are not conclusive. A. J. Durelli, USA

682. M. Roš, Fatigue strength of welds (in French), Rev. Metall. 45, 421–446 (Nov. 1948).

This paper summarizes results of a great deal of work, at the Swiss Federal Testing Laboratory (LFHM), on fatigue strengths of welded joints in structural steel.

Part I gives fatigue limits (at one million cycles, under zero-to-tension loading) for a number of types of welded joints. While the tests are not described in great detail, there are several photographs of failed specimens and considerable information, scattered through the paper, concerning the materials and test pieces used.

Part II outlines the author's method of interpreting fatigue strengths in terms of a "generalization of the Mohr theory" of strength, the idea being that fatigue strength is dominated by a critical limiting shear stress in the octahedral plane of the elementary cube. Some readers will not agree that this theory is sufficiently well confirmed to justify detailed use in interpretation of fatigue strength of joints. Nevertheless, this concept is here used as a framework for assembly of test results, and for suggested design calculations.

Later parts suggest permissible strength values for use in design of structures with welded joints. Considerable attention is given to joints under biaxial stress loading. Effects of machining weld beads and of stress-relief heat treatments are also shown in several diagrams.

Final discussion includes fairly complete references to previously published articles, including LFHM reports, on which this summary account is based. H. Grover, USA

Structures

(See also Revs. 660, 661, 679)

683. Leroy A. Beaufoy and A. F. X. Diwan, Analysis of continuous structures by the stiffness factors method, Quart. J. Mech. appl. Math. 2, no. 3, 263–282 (Sept. 1949).

The authors have presented a method of analysis of continuous structures in which the solution of lengthy simultaneous equations has been avoided and which is applicable to such complex structures as continuous arches on elastic supports.

In the analysis, the structure is first divided up into units which are considered as fixed-ended. The forces and moments required to produce unit deformations and rotations at the various joints are then computed. The joints are released, and the forces and moments required to produce unit rotations and deflections at the joints are computed together with movement of the joints induced by movements of other joints.

After computing the above elastic coefficients, the structure is loaded with the design loads, and the forces and moments are computed at each joint assuming all joints to be fixed. The unbalanced moment and forces at each joint are then determined, after which the joints are released and the movement of the various joints due to the unbalanced moment and forces at each joint is determined, together with the movement induced by the movement of other joints. Once having the total movement at each joint the various moments and forces acting at each joint can be readily computed from the previously determined stiffness factors.

The authors solved a problem of a three-span arch on elastic supports with a uniform load in one span. The results check those obtained by a method of successive approximation.

The method is applicable to frames containing curved members with varying cross section, and takes into account the effect of side sway. All computations can be made by slide rule. Karl Arnstein, USA

684. Hjalmar Granholm, On composite beams and columns with particular regard to nailed timber structures (in Swedish), Trans. Chalmers Univ. Technol. Sweden no. 88, 214 pp. (1949).

The paper deals with an approximate theoretical and an experimental analysis of the influence of joint flexibility in composite beams and columns, glued or nailed joints, etc. The joint flexibility is determined by a factor K (k), being the ratio of shear stress (shear load per nail, bolt, etc.) to the relative displacement of the joined components. The approximative nature of the theoretical analysis consists of the assumption that K or k is a constant, whereas tests on timber connections generally show a large decrease at increasing load. The study appears to be of importance mainly for built-up timber structures (various current design specifications being criticized), but may also be of practical significance for other cases incorporating flexible joints (e.g., glued metal-to-metal joints).

After an introductory section containing some numerical values of k as a function of shear load for various timber connections, there follow 8 sections dealing with the deflections, joint displacements, joint shear loads and bending stresses of beams built

up of two or three components. It is shown, both theoretically and experimentally, that these may show large deviations in magnitude and distribution from those following from the ordinary beam theory.

Of the following four sections two contain an analysis of the stress distribution in and the stiffness of long glued or nailed joints with some applications, also to reinforced concrete structures and piles. The other two are on latticed beams and Vierendeel girders. In all cases the agreement between theory and experiments is reasonable. The final eleven sections are concerned with buckling of composite struts, latticed columns and Vierendeel columns. The theory is based on equations derived in previous sections, and in the experiments the buckling load is determined by Southwell's method. The tests show a large influence of the unpredictable and time-dependent friction in the joints and of initial eccentricities on the failure load. Special tests were made employing latticed columns having frictionless joints. Some tests also show a marked reduction of the failing load as the load is sustained during a long period.

F. J. Plantema, Holland

685. **N. J. Hoff and Paul A. Libby, Recommendations for numerical solution of reinforced-panel and fuselage-ring problems,** Nat. adv. Comm. Aero. tech. Note no. 1786, 79 pp. (Dec. 1948).

In applying the indirect approach, operations tables are first established in accordance with R. V. Southwell's method. The operations tables along with the external forces constitute a system of linear equations which are equal in number to the degrees of freedom of the structure. The displacements can be found from these equations by various approximate methods.

Among these methods Southwell's method of systematic relaxation is applied to the reinforced-panel problem and recommendations are made for obtaining rapidly converging solutions. The matrix calculus method is illustrated and recommended, instead of Southwell's relaxation method, for the cases where the number of equations is greater than 30 or 40 and/or the panels have sheets with large shearing rigidity.

The growing-unit method, Niles tables and electric-analog methods are treated, and some of them are illustrated with applications to problems of fuselage rings with various shapes. The authors conclude that: (1) in most reinforced-panel problems the use of the relaxation procedure is advantageous; (2) solution of the equations defining a reinforced-panel problem by means of the electric analog is advisable when many closely related problems have to be investigated; (3) ring problems are best solved by matrix methods; (4) in very complicated ring problems a combination of matrix methods with the growing-unit and relaxation methods may become advisable.

A. Cemal Eringen, USA

686. **Herman Amos and Walter Bochmann, Test to determine the interaction of prefabricated elements of reinforced concrete for floors** (in German), Dtsch. Ausschuss Stahlbeton no. 101, 59–84 (1948).

Prefabricated reinforced-concrete floors, consisting of reinforced-concrete beams 75 cm center to center and nonreinforced slabs 32 cm wide resting on the beams, were tested for ultimate load when the joints between beams and slabs were either dry or connected by mortar, and slabs were connected to the beams by steel rods 30 cm apart or not so connected. It was found that the provision of such steel cross connection increases the ultimate load by 40%, while mortaring-in of the joints gives an increase of 7%. German regulations therefore require for prefabricated slabs the provision of such connecting rods at right angles to the beams.

M. Reiner, Israel

687. **Allan Joshua Ockleston, The effect of static and dynamic loads on a reinforced-concrete tied-arch bridge,** J. Instn. civ. Engrs. 32, 50–79 (Mar. 1949).

Although this investigation was made primarily to prepare the design of a bridge on the Umhlatuzi River in Zululand, the final results have a much wider range of application. Impact allowance had been made in the design, but it was found desirable to check this allowance by conducting tests on a model which was structurally similar to the proposed bridge. General methods of design were then applied to the model as built and results checked experimentally to study the validity of different assumptions.

In the mathematical analysis of any tied-arch bridge, the chief uncertainty being the interaction between the deck system and the arch rib, five methods of calculation were tried, starting with the assumption that the deck system and the hangers were of negligible stiffness and increasing the importance of that deck stiffness to the maximum possible. Comparison of the calculated deformations with the measured ones were then made for two types of loading. In the same manner, the natural frequency of vertical vibrations was obtained by purely analytical means and compared with the measured value. Lateral vibrations were measured experimentally only, and so was damping, both values being applied directly to the design of the structure. Forced vibrations such as may be caused by locomotives were also studied, and finally some tests were run on the prototype before the bridge was put into service, as a check on the model investigation.

A number of conclusions are drawn among which are: the deck system in a tied-arch bridge acts as a single member in a way similar to that of a stiffening girder in a suspension system; furthermore, the approximate method of analysis overestimates the rib moments for concentrated loads, and there is justification for employing in this method a larger value for the flexural rigidity than is customary. Finally, natural frequencies of vertical vibration and lateral vibration of the hangers can be estimated by approximate methods which are rapid and sufficiently accurate for most purposes, but although amplitudes can also be calculated, the method is too laborious for practical purposes.

Robert Quintal, Canada

688. **Otto Graf and Kurtz Walz, Tests to determine the crack formation and strength of reinforced concrete slabs with various reinforcing steels under gradually increasing loads** (in German), Dtsch. Ausschuss Stahlbeton no. 101, 1–39 (1948).

Steel rods of different cross-sectional shapes with and without end hooks were used as reinforcement to concrete slabs. It was found that there was no advantage in using rods of noncircular cross section, but that a rough surface of the rods is of advantage. Hooks do not add to either ultimate strength or safety against cracking. When bent-up bars are provided, the slabs fail through excessive extension of the steel in the tensile zone. When the reinforcement is straight, the slabs fail through the appearance of sloped tension cracks in regions between the point of application of load and the support, accompanied by slippage of steel, with steel stresses below those when bars are bent up. Bending up bars is economical.

M. Reiner, Israel

Rheology (Plastic, Viscoplastic Flow)

(See also Rev. 681)

689. **Hans Stäger, General technology of materials (Allgemeine Werkstoffkunde),** Verlag Birkhäuser, Basel, 1947, 423 pp., 296 figs. Cloth, 9.5 × 6.5 in., approx. $11.

The demands made by modern technology on the properties of materials are becoming increasingly severe, and the situation

can be alleviated only if the producers of materials (the chemical and metallurgical engineers) are deeply aware of the nature of these demands, while the users (the design engineers) on the other hand are well informed of the properties of the available materials. The purpose of the present book, as stated by the author in the preface, is to further understanding between these two major professional lines by presenting a comprehensive review of existing knowledge on the structure and behavior of engineering materials.

The book is divided into thirteen chapters, the contents of which may be briefly indicated as follows: (1) Introductory remarks on the role of materials in modern civilization; (2) Methods of analysis, atomic bonds, formation of molecules; (3) States of aggregation, crystalline, amorphous and glass structures; (4) Grain formations, textures (discussion of bearing metals); (5) Cold working, recovery and recrystallization; (6) Allotropy, iron-carbon and copper-zinc equilibrium diagrams; (7) Elastic and plastic deformations, conditions of fracture, time effects; (8) Impact and fatigue strength, thermal effects; (9) Dielectric strength and its relation to mechanical properties; (10) Corrosion of metals; (11) Behavior of heterogeneous systems, gaseous-liquid-solid, under alternating loads and in electric fields; (12) Biological deterioration of matter; (13) Corrosion of organic materials.

On account of the large field covered, the book has necessarily an encyclopedic character, and proofs or detailed discussions of phenomena are omitted. However, the book is well documented with references and, on the whole, it serves excellently its purpose of giving an over-all view of the present state of basic knowledge on engineering materials. M. Hetényi, USA

690. S. I. Gubkin, Methods of determining deformability (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk no. 9, 1463–1482 (Sept. 1948).

The author proposes to classify the "mechanical states" of an incompressible plastic material according to the signs of the principal stresses $\sigma_3 \leq \sigma_2 \leq \sigma_1$, the mean normal stress $\sigma = (\sigma_1 + \sigma_2 + \sigma_3)/3$, and the ratio $\nu = [2\epsilon_2 - (\epsilon_1 + \epsilon_3)]/(\epsilon_1 - \epsilon_3)$, where $\epsilon_3 \leq \epsilon_2 \leq \epsilon_1$ are the principal strains. Three types of strain, namely: (a) $-1 \leq \nu < 0$, (b) $\nu = 0$, (c) $0 < \nu \leq 1$, and five types of stress, namely:

$$(\alpha) \quad \sigma_1 \leq 0, \sigma_3 < 0, \sigma < 0, \sigma_2 \leq 0,$$
$$(\beta) \quad \sigma_1 > 0, \sigma_3 < 0, \sigma < 0,$$
$$(\gamma) \quad \sigma_1 > 0, \sigma_3 < 0, \sigma = 0,$$
$$(\delta) \quad \sigma_1 > 0, \sigma_3 < 0, \sigma > 0,$$
$$(\epsilon) \quad \sigma_1 > 0, \sigma_3 > 0, \sigma > 0, \sigma_2 \geq 0,$$

combine to give fifteen types of mechanical states.

The author then introduces, in a rather arbitrary fashion, an "index of mechanical state," a single number purporting to characterize the mechanical state. Finally, the difficulties which arise in the classification of nonhomogeneous states of stress and strain are discussed. W. Prager, USA

691. V. M. Panferov, On the convergence of the method of elastic solutions in the theory of elastoplastic shell deformations, (in Russian), Prikl. Mat. Mekh. 13, 79–94 (1949).

Following A. A. Ilyushin (*Plasticity*, Moscow, 1948) the author deduces the differential equations of the axisymmetric elastoplastic deformations of a cylindric shell. There results a differential equation of fourth order for the radial displacement of a medium plane, which is linear in the elastic terms but nonlinear in the plastic ones. This equation can be transformed into an integrodifferential equation. The first approximation, ignoring the plastic terms, is composed of linear operators only. The method of elastic solutions consists now of iterative approximations, the convergence of which is shown by the author, after proving the existence and uniqueness of solutions.

Walter Wuest, Germany

692. Ya. L. Lunts, On the propagation of spherical waves in an elastoplastic medium (in Russian), Prikl. Mat. Mekh. 13, 55–78 (1949).

The propagation of a pulse of pressure acting on the surface of a spherical cavity in an elastoplastic medium is considered. A work-hardening law involving a general relation between the second-order invariants of the stress and strain deviators (i.e., the expression for the von Mises distortional strain-energy yield criterion, and the similar expression in strain components) is assumed; a deformation type of law which in this case includes a flow type of law because of the symmetry of the system. The equation of motion is developed according to the usual assumptions of small displacements, and it represents both elastic and plastic spherical waves. The method of analysis for the wave-front form is developed by means of the theory of characteristics, the independent variables r, radius, and t, time, being transformed to variables constant along the characteristics.

The propagation of the wave front due to a given pulse of increasing pressure is determined for a linear work-hardening law. The propagation of an elastic unloading wave due to a subsequent decrease of the pressure is also analyzed.

E. H. Lee, USA

693. Ruth Moufang, Rigorous calculation of residual stresses remaining in plastically expanded tubes after discharging of the pressure (in German), Z. angew. Math. Mech. 28, 33–42 (Feb. 1948).

A theory is considered which the author uses for the computation of the stresses in a thick-walled hollow cylinder under an internal pressure and for plastic strains. The theory is based on the law of strain hardening of ductile metals due to R. Schmidt (Göttingen), according to which the second invariant of the stress tensor τ_0 is a function of the second invariant of the deviator of the permanent strains γ_0,

$$\tau_0 = f(\gamma_0),$$

where

$$\tau_0 = \sqrt{\tfrac{2}{3}} \cdot \sqrt{\sigma_r^2 + \sigma_t^2 + \sigma_z^2 - \sigma_r\sigma_t - \sigma_t\sigma_z - \sigma_z\sigma_r}$$

and

$$\gamma_0 = \sqrt{\tfrac{2}{3}} \cdot \sqrt{\epsilon_r^2 + \epsilon_t^2 + \epsilon_z^2 - \epsilon_r\epsilon_t - \epsilon_t\epsilon_z - \epsilon_z\epsilon_r}.$$

The author considers only the special case of a cylinder in which the axial strain ϵ_z vanishes during the loading and the unloading process of the tube (σ_r, σ_t, σ_z are the radial, tangential, axial stress; ϵ_r, ϵ_t, ϵ_z the corresponding permanent strains). As strain-hardening function $f(\gamma_0)$, the author assumes an inclined straight line in the plastic range of strains as a special case. During the unloading process $\tau_0 = f(\gamma_0)$ is assumed as a steeply inclined straight line parallel to the one corresponding to Hooke's law. Under these assumptions explicit formulas for the stresses σ_r, σ_t, σ_z are developed in the cylinder, yielding under an increasing pressure p and upon a gradual release of p until $p = 0$. These formulas determine the distribution of the residual stresses after the release of the pressure in the tube. If Hooke's law is assumed during the release of the internal pressure p, the same result for the residual stresses would have been obtained by super-

posing the elastic distribution of the stresses in a thick-walled cylinder (the Lamé formulas) on the distribution of the stresses at which yielding was interrupted, and by assuming $\sigma_r = +p$ (equal to a tensile stress p) at the inner surface of the tube for this elastic state of stress. The author, however, considers also more general unloading curves showing the Bauschinger effect.

A. Nadai, USA

694. **W. R. Dean and E. H. Mann, The change in strain energy caused by a dislocation,** Proc. Cambridge phil. Soc. 45, 131–140 (1949).

A dislocation of the type suggested by Bragg is treated. The center of dislocation (a pair of circular cylinders joined by a slit) is excluded from the calculations. [Bragg treats the case where the center of dislocation is assumed to have the energy of latent heat; same source, 125–130 (1949).] The stress function and the displacements for two plane strain problems are given: (1) simple shear; (2) a Bragg-type dislocation. The difference in the strain energy of system (1) and that of a system comprised of "(1) plus (2)" depends upon the shape of the outer boundary; for a circular outer boundary the strain energy of the combined system is less than that of the first system alone. The conclusion is drawn that a dislocation of amount s may be expected to form whenever the shear is greater than $(2s/t) \log (t/R)$, where t is the distance between the axes of the cavities and R is the common radius.

Courtesy of Mathematical Reviews R. C. Meacham, USA

695. **N. F. Mott and F. R. N. Nabarro, Dislocation theory and transient creep,** Rep. Conf. Strength Solids, Univ. Bristol, July 1947, Phys. Soc. Lond., 19 pp. (1948).

The authors begin with a brief review of the various developments in dislocation theory. They point out that it is now usual to base any theory of strength on the assumption that a perfect crystal is very strong, the stress necessary to produce simultaneous slip of one layer of atoms over another being orders of magnitude greater than measured yield points. Most modern theories of slip assume that slip begins at one end of the crystal and travels across it, avoiding the concept of simultaneous slip. The discontinuity existing when slip has progressed partially across a crystal is called a dislocation. Some of the properties of the dislocations of the edge type are mentioned, including definition of positive and negative dislocations, attraction of dislocations of opposite sign, and the role of screw dislocations. The theory of solids has been far enough advanced to indicate that, for a simple cubic lattice, the stress necessary to move a dislocation in an otherwise stress-free body is of the order of 1/2000 of the shear modulus, and may be as low as 10^{-7} to 10^{-10} times the shear modulus for other lattice configurations. It is also known that it is out of the question for dislocations to arise thermally at any temperature at which the substance is solid.

It is pointed out that the experimental facts indicate that hardening of materials is related not to the formation of more dislocations, but rather to the difficulty of moving them. A brief description of the Taylor theory of work hardening is given. From this theory it is concluded that experiments on single crystals or pure annealed materials must be interpreted in terms of motion of dislocations in a stress field of potential which varies not only with hardening but with the applied stress. The authors therefore propose studies of creep and slip in an age-hardened material in which aging occurs only at a temperature higher than that of the test.

The remainder of the article is devoted to a discussion of the deformation of this type of material and solid solutions. Mathematical developments of the shape of dislocations in a stress field and their motion are given, the latter for very wide dispersion of precipitate or for solid solutions.

An extensive development of transient-creep relations is presented which leads first to the conclusion that the yield stress (defined in terms of strain rate) is nearly independent of temperature unless it is assumed that the internal stress is a function of temperature. The theory as developed is one of "exhaustion" creep such as is shown by lead at temperatures of −180 C and −78 C. A similar theoretical development is made of slip in solid solutions. This results in relations of change in yield strength to concentration and to change in lattice parameter which are in approximate agreement with experiments.

The paper is very complete and points the direction for much further work. Very little of the notation is defined, so that the mathematics will be largely unintelligible without careful reference to a previous paper by Nabarro [Proc. phys. Soc. Lond. 59, p. 256 (1947)]. Morton B. Millenson, USA

Failure, Mechanics of Solid State

(See also Revs. 681, 682, 688, 689, 693, 695)

696. **Hayo Föppl, Measurement of stresses and plastic deformations of test bars subjected to surface rolling** (in German), Mitt. Wöhler Inst. Braunschweig no. 41, 3–67 (1948).

The fatigue strength of a cylinder, bolt, spring, etc. is greatly increased by surface rolling. During this process the boundary layer of the material deforms plastically, and this results in a state of stress for the whole of the specimen, showing itself as an elongation. In studying these phenomena the author makes use of cylindrical test bars with a diameter of 44 mm and a length of 200 mm. By the aid of a special device the changes of length of the test bar could be measured accurately. The plastically deformed surface layer is gradually removed by etching the bar in diluted nitric acid. In completing this etching the bar regains its old length. The thickness of this layer is determined from the loss of weight. The changes in length of the test bars are plotted against the according thickness of the dissolved layer in the so-called "etching-diagram." From it the axial stress in the surface layer may be calculated. As only the axial strain could be measured satisfactorily, the value of the tangential strain is estimated in the assumption that under the same testing conditions there exists a fixed relation between the axial and tangential strain, $\epsilon_t = \delta\epsilon_a$. It is shown that the slope of the etching-diagram is a measure for the value of the axial stress in the surface layer. The second part of the book deals with the results of a number of tests on test bars from spring steel and mild-alloy steel. The influence of the roller pressure, roller feed, and number of passages is discussed. R. G. Boiten, Holland

697. **H. Moore, The fracture of solids, part 2,** Metallurgia 39, 179–181 (Feb. 1949).

The author presents a brief résumé of the current general knowledge of the fracture of glass, giving a brief discussion of Griffith's crack theory, the usually noticed phenomena of fine filaments and moisture effect, and the discrepancy between computed and actual strengths. Morton B. Millenson, USA

698. **René Castro and André Gneussier, Analogies between factors causing brittleness of polycrystalline metals** (in French), C. R. Acad. Sci. Paris 228, 1339–1341 (Apr. 20, 1949).

This is a study of three factors causing brittleness in steels, and a generalization of their similar effects from consideration of the specific energy of rupture, and how it varies with the several factors. Bending impact tests on notched specimens were used. The factors of embrittlement are triaxiality, the reciprocal of

the temperature, and the speed of deformation. Triaxiality is considered as increasing with the dimension perpendicular to the profile, but it is not given a really adequate definition.

The claim is made that for these three factors the following is true: (1) If any two of the three factors of embrittlement remain constant, the curve of specific energy of rupture passes through a maximum as the third increases, and beyond this maximum occurs the transition to brittle fracture. (2) If any one of the factors is held constant, the zone of transition to brittleness and the maximum in the curve of energy are displaced toward decreasing values of a second factor as the third factor increases. W. C. Johnson, Jr., USA

699. John Straub, The importance of uniformity in the application of the shotpeening treatment, Soc. auto. Engrs. J. 56, 37–38 (Nov. 1948).

The author emphasizes his conclusion, based on a number of convincing test results, that uniformity in size of the shot particles is very important when low cost of the shot-peening treatment is aimed at. In relation to the particles of nominal size the broken ones are completely ineffective with respect to the increase of the endurance limit of shot-peened parts, on account of their reduced mass. The author's statement implies not only that broken shot should be removed from the peening machine continuously and as effectively as possible, but also that the shot should be supplied with the minimum practicable variation in size. J. A. Haringx, Holland

700. F. P. Zimmerli, Results of tests made to ascertain effect of various kinds of shot on fatigue life, Soc. auto. Engrs. J. 56, 36–39 (Nov. 1948).

Tests with shots of various hardness have ascertained that the normal increase of the endurance limit of shot-peened parts can also be obtained by soft shots (steel or heat-treated white cast iron). However, both the appearance of the surface and the Almen arc height deviate considerably from usual specifications, and are evidently no reliable indications of the effectiveness of the shot-peening treatment. The shot life appears to be inversely proportional to the hardness of the shot.

Further tests have shown a small tendency to overpeening in the case where a too coarse shot had been used. For the rest the size of shot has no effect upon the endurance limit.

J. A. Haringx, Holland

Design Factors, Meaning of Material Tests

(See Revs. 682, 686, 688, 696, 704, 706)

Material Test Techniques

(See also Revs. 681, 696, 708, 711, 768)

701. Johannes Perthen, Testing and measurement of surface shapes (Pruefen und Messen der Oberflaechengestalt), Carl Hanser Verlag, Munich, 1949, 257 pp., 115 figs. Paperboard, 9.5 × 6.5 in., approx. $6.

The book discusses thoroughly all aspects of the observation, measurement, and description of the geometric form of surfaces which is, according to the author, only one of the three factors that must be judged separately in order to determine the true quality of a surface. He expresses hope that later texts in this series of monographs will deal as completely with the condition of surfaces (their physical and chemical nature, hardness, etc.) as well as their behavior (change in physical condition, energy, mass, work hardening, and corrosion).

Essential surface-geometry measurement knowledge and progress in Germany, United States and England for the last 20 years are described in detail with a satisfying thoroughness typical of a German scientist, in an effort to provide practical help to designers and foremen. The author admits that subject to be in a state of flux due to the complex nature of surface forms, as well as to difficulties in developing suitable instrumentation and a satisfactory theory upon which a workable and practical three-dimensional measurement system can be based. It is stated that in surface checking (forever perhaps) emphasis will be on comparative measurement in contrast to precise numerical measurement. Although many still consider surface measurement and designation a luxury, he states that its introduction has always led to reductions in costs or improvement in products. He hopes, by this text, to bring to each operator, measurement engineer, designer, those portions of the field that are of interest, thereby reducing the uncertainty of measurement and at the same time showing its practical limitations.

James A. Broadston, USA

702. G. A. Cottell, K. M. Entwistle, and F. C. Thompson, The measurement of the damping capacity of metals in torsional vibration, J. Inst. Metals 74, 373–424 (Mar. 1948).

The authors investigated various sources of discrepancies between the results of "mechanical" and "physical" methods of measuring damping capacity in torsional vibration. In the physical method a specimen in the form of a uniform cylindrical bar is vibrated in the free-free mode, while in the mechanical method the frequency of vibration is much reduced by fixing two inertia masses on the ends of the specimen. In the latter method the energy loss in the machine itself may be a considerable portion of the total energy loss measured for machine and specimen. It is reported that for a duralumin specimen in which damping is small, the energy loss in a machine of the Föppl-Pertz type was about 500 times the energy loss of the specimen. The major sources of machine loss were isolated and the machine was completely redesigned. The damping capacity obtained by means of this machine was in the order of two times the lowest values measured by the physical method. A small machine also was constructed for conducting tests at reduced air pressures. It is reported that the damping capacity of duralumin as obtained by means of this machine is in excellent agreement with the lowest determinations by physical methods.

A procedure is presented in the appendix of the paper for obtaining by means of experimentally recorded decay curves an estimate of the specific damping capacity at any measured amplitude. Frank Baron, USA

703. B. G. Heebink and A. A. Mohaupt, Investigation of methods of inspecting bonds between cores and faces of sandwich panels of the aircraft type, For. Prod. Lab. Rep. no. 1569, 1–28 (Sept. 1947).

704. H. R. Copson, Factors of importance in the atmospheric corrosion testing of low-alloy steels, Proc. Amer. Soc. Test. Met. 48, 591–609 (1948).

Some complications in the atmospheric corrosion testing of low-alloy steels are brought out. Data are presented which show that it is unsafe to draw general conclusions from tests of limited character. The results obtained depend on the location (whether industrial, marine, or rural), on the duration of the tests (the benefit of alloying increases with time), on the manner of exposure (particularly whether sheltered or boldly exposed), on the method of estimating corrosion (whether weight loss, pit depth, or time to disintegration), and on weather (particularly as re-

gards wetness and pollution). Under all conditions alloying seems of value, but the magnitude of the benefit and the relative merit of different steels and alloying elements vary. For example, a few weight-loss vs. time curves have been found to cross. In a thin steel, pit depths become more important than weight losses. Likewise for short exposures the benefit to be obtained by complex alloying may not be worth the additional cost. These effects are illustrated chiefly by means of data obtained through five years of exposure of 71 low-alloy steels at a marine location at Block Island, R. I., and an industrial location at Bayonne, N. J.

Author's summary

705. G. A. Ellinger, L. J. Waldron, and S. B. Marzolf, Laboratory corrosion tests of iron and steel pipes, Proc. Amer. Soc. Test. Mat. 48, 618–627 (1948).

Continuous-flow laboratory corrosion tests of ten types of uncoated ferrous pipes were made over periods of time extending up to ten years. The corroding medium was Washington, D. C., tap water continuously circulated through columns of the test samples. Corrosion was measured as loss of weight and depth of pits in relation to the time of exposure. While most of the pipes corroded at somewhat similar rates, there were small but real differences in the corrosion of some of the materials.

Authors' summary

706. Robert C. McMaster, An investigation of the possibilities of organic coatings for the prevention of premature corrosion-fatigue failures in steel, Proc. Amer. Soc. Test. Mat. 48, 628–648 (1948).

Repeated stressing of steels exposed to corrosive brines often causes premature corrosion-fatigue failures. A few readily available organic coatings were investigated as possible means of protection. The results of the laboratory tests were confirmed by field experience in the oil-well drilling industry.

Room-temperature R. R. Moore rotating-beam fatigue tests were made of bare specimens of SAE 1045 steel operating in air and in a corrosive brine containing 7% total salts. Single specimens with intact and with scratched organic coatings were tested in the brine.

The hot-rolled and normalized steel had a static ultimate tensile strength of 107,000 psi. The yield strength was 62,500 psi. The endurance limit in air under reversed bending stresses was about 48,000 psi. The corrodent had little effect upon the 1750 cycle-per-min operating life of bare steel, at stresses above 48,000 psi. Below this stress level, the corrosion-fatigue life of bare specimens increased about 10 times for each 20,000 psi reduction in the applied stress.

Organic coatings significantly lengthened the corrosion-fatigue life at stresses below 48,000 psi in these tests. A typical four-coat system increased the operating life at 40,000 psi maximum stress from 430,000 cycles (bare) to more than 64,000,000 cycles (coated). This coating contained 60% alkydtype and 40% urea-formaldehyde type resin solids as a binder. It contained a protective pigment in the primer coats, and was baked at 250 F after each coat. A five-coat system of air-drying materials increased the operating life under the same test conditions to more than 37,000,000 cycles.

Several additional complete organic coating systems were prepared. These specimens have not yet been tested. There is little hope of causing these coatings to fail under the standard test conditions (40,000 psi maximum reversed stress, 1750 cycles per min) in periods shorter than several months.

From these tests it appeared possible that intact organic coatings might provide great help in preventing premature corrosion-fatigue failures of steels in industrial equipment subjected to corrosive environments. However, after failure of the organic coatings (simulated in the tests by scratches cut through the coatings), the remaining operating lifetime was found to be about the same duration as that of new, bare specimens.

Author's summary

707. N. B. Pilling and W. A. Wesley, Atmospheric durability of steels containing nickel and copper—additional exposure data, Proc. Amer. Soc. Test. Mat. 48, 610–617 (1948).

The prior publication (same authors, same source, 40, p. 643, 1940) on this subject appeared in 1940 and described six series of weathering tests designed to evaluate the durability of sheet specimens of low-alloy steels containing nickel and copper. The present communication brings up to date the published results of those exposure tests which were continued beyond 1940. In the oldest series, observations at the end of 22 years' exposure are now made available. In the series in which large sheet specimens simulate some of the conditions encountered in roofing service, the advantage of nickel-copper steels over copper steels is becoming more impressive as the years pass by. Long-continued tests show that it is more significant to compare steels on the basis of how long a time is required to arrive at a given unit weight loss, than to compare them on the basis of weight loss suffered in a fixed period of time.

The beneficial effects of nickel upon the weathering resistance of steel increase with increasing nickel content. Addition of copper to a nickel steel improves the life, but the optimum amount is less than the amount of nickel and remains well below 2% even with a 4% nickel content. With long exposure periods, there is evidence that not only phosphorus but also silicon additions to nickel-copper steels have a beneficial effect on behavior in the industrial atmosphere. Effects of manganese and carbon are unimportant in this type of atmosphere.

Authors' summary

Mechanical Properties of Specific Materials

(See also Revs. 662, 686, 688, 689, 697, 698, 704, 707, 779, 780, 781, 782, 783)

708. R. F. S. Hearmon, The elasticity of wood and plywood, For. Prod. Res. spec. Rep. no. 7, 1–87 (1948).

The author presents a review of theoretical and experimental work carried out on the elastic properties of wood and plywood through 1946. The subject matter is divided into three parts: (1) the elastic constants of wood, (2) the elastic constants of plywood, and (3) the elastic properties of plywood plates and cylinders.

In addition to discussing the elastic constants of wood in part I, various methods of obtaining these constants and the relative merits of each test are presented. The effect of such factors as frequency (of vibratory tests), temperature, and moisture content on the elastic properties are discussed in detail. The mathematical theory of wood elasticity is presented and the theoretical results are compared to experimental.

Part II covers the development of the elasticity of plywood. Again both theoretical and experimental investigations are compared.

With the theoretical and experimental elastic properties of plywood presented, the author then presents some work on two-dimensional stress problems. The general theory for plywood plates is developed first and then applied to problems of the buckling of plywood plates, the vibration of 0-deg and 90-deg plates, the deflector of 0-deg and 90-deg plates under lateral load, and finally the buckling of plywood cylinders.

While some of the material presented is original, the bulk of the information is assembled from papers published in the USA, Great Britain, France, Germany, and the British Dominions. The work is well annotated, the bibliography including 140 references.

Frank J. Mehringer, USA

709. **R. Potaszkin, On mechanical properties of forged pieces of steels with small contents of nickel, chrome, and molybdenum** (in French), Rev. Metall. 64, 125–140 (Mar. 1949).

As a result of tensile tests on 1500 specimens ranging from 1 to 8 in. in diam, the author reports on the properties of steel forgings of compositions similar to United States grades of steel NE 8627, NE 8735 and NE 9945. The effects studied are: size of specimen, location of material of specimen in forging, heat treatment, and mode of quenching. Photomicrographs are presented to illustrate the grain structure of the various specimens. Among the properties reported are the elastic limit, ultimate strength, per cent elongation and reduction of area. Properties of common European steel alloys of Ni-Cr, Cr-Mo, Ni-Cr-Mo, Cr-Mn, and Cr-Mn-V, obtained and reported by another laboratory, are used by the author as a basis for classifying each NE steel with these five alloy steels as regards their elastic limit and resilience when treated to the same ultimate strength. These classifications show that the NE steels are slightly weaker than the high-alloy-content steels of Cr-Mo, and Ni-Cr-Mo, but they compare favorably with the alloy steels of Ni-Cr, Cr-Mn and Cr-Mn-V.

Alexander Yorgiadis, USA

710. **W. L. Fink and L. A. Willey, Quenching of 75S aluminum alloy,** Trans. Amer. Inst. min. metall. Engrs. 175, 414–427 (1948).

The material under consideration is a high-strength aluminum alloy containing 5.7% Zn and smaller amounts of Mg, Mn, Cu, and Cr. Tests indicated that the cooling rates of alcohol at 70 F, boiling water, air blast and still air averaged about 86, 42, 13 and 4 F per sec respectively. These media were used in a delayed quenching procedure and in an interrupted quenching procedure. Greater rates were obtained with a simple quenching procedure using water at 70 F (2500 F per sec), Woods metal at 170 F (1900 F per sec), light oil (1600 F per sec), medium oil (275 F per sec) and heavy oil (128 F per sec). Tensile tests on sheet and extruded rod indicated that the tensile and yield strengths of 75S-W and 75S-T remain at the maximum level when the quenching rate exceeds 800 F per sec. Rates as low as 200 F per sec are only slightly inferior. The resistance to corrosion of 75S-T in alternate immersion in salt water is not materially impaired by quenching rates as low as 200 F per sec. At slower rates the resistance decreases rapidly to a minimum at about 35 F per sec. At still slower rates it increases. The results are explained.

Marshall Holt, USA

711. **L. R. G. Treloar, Stresses and birefringence in rubber subjected to general homogeneous strain,** Proc. phys. Soc. Lond. 60, 135–144 (Feb. 1948).

In modern engineering the uses and applications of rubber are constantly growing. The knowledge of the stress-strain relationship of rubber is still insufficient. The author has performed a number of tests with rubber sheets subjected to the most general type of homogeneous strain by loading them with two independent sets of forces at right angles. The occurring birefringence was measured in terms of the wave-length of sodium by the aid of a polariscope and a Babinet compensator. The strains were determined from the deformation of a square lattice, made by drawing lines on the surface of the sheet.

As predicted by the molecular network theory (long-chain molecules and randomly-jointed links) the birefringence would be proportional to the difference of the squares of the principal extension ratios and to the difference of the principal stresses in the sheet. This statement was in good accordance with the experimental results, especially with sheets swollen in paraffin. The principal stresses t in the plane of the sheet may be expressed as functions of the principal extension rates λ as:

$$t_1 = G(\lambda_1^2 - \lambda_3^2), \quad t_2 = G(\lambda_2^2 - \lambda_3^2).$$

Under the assumption that there is no change of volume on straining, these equations were checked with the experimental data. For both the dry and the swollen rubber the agreement was poor.

After a review of some possible explanations for this evidence, the equations of Mooney and Rivlin for the work of deformation, based on an assumed stress-strain relationship, are compared with the experimental results. With swollen rubber, there is good agreement with the Mooney equation, but for dry rubber it seems to be necessary to add further terms to this equation.

R. G. Boiten, Holland

Mechanics of Forming and Cutting

(See Revs. 696, 699, 700, 701)

Hydraulics; Cavitation; Transport

(See also Revs. 767, 768, 777)

712. **Charles Jaeger, Technical hydraulics** (**Technische Hydraulik**), Verlag Birkhäuser, Basel, 1949, 464 pp., 303 figs. Cloth, 6.7 × 9.6 in., $11.

This book gives a complete presentation of everything needed for the planning of hydraulic power stations of any kind (high, medium and low-head power plants). Its basic ideas are the general laws of energy and impulse and the laws of similarity of Reynolds and Froude. It makes use of all those conceptions, definitions, theoretical, semitheoretical and experimental methods and results of modern hydro- and aerodynamics which may be applied to technical hydraulics in field of water-power plants.

The book presents the physical foundations of hydraulics, derives the equations of Euler, Lagrange, Navier-Stokes and Bernoulli, and the law of momentum specialized to fluid mechanics. After a discussion of potential, rotational, laminar and turbulent flow, it offers an ample treatment of the steady flow through pipe lines and open channels—continuous and discontinuous ones. Then unsteady flows are considered, and oscillations in surge tanks, the water-hammer phenomenon and the hydraulic jump in open channels are thoroughly treated. Finally seepage through porous materials is analyzed. In two appendixes many experimental data and their applications to practical problems are given. A short treatment of transport of sediments is included.

The book also presents many practical, graphical and semigraphical methods. It puts technical hydraulics on an exact, rational basis, explaining the basic ideas clearly and making them applicable to practical problems.

The text is supplemented by a complete bibliography in numerous footnotes, which give an interesting picture of the development of the science of hydrodynamics and hydraulics.

W. Spannhake, USA

713. G. A. Oosterholt, An investigation of the energy dissipated in a surface roller, Appl. Sci. Res. Sec. A, no. 2, A 1, 107–130 (1948).

In a first, rather theoretical part of the paper, the author develops a general form of the energy equation in differential form which expresses the fact that the decrease of the mechanical energy according to the Bernoulli equation, plus the work done by the internal-stress tensor in the fluid, must equal the rate of energy W which is transformed into heat. Then he proceeds to calculate this value W, the energy loss. In setting up the stress tensor, he includes turbulence terms of the form $\tau = \overline{u'v'}$, only to neglect them in the next line; with this procedure, he actually includes all turbulence energies in W without saying so. After neglecting some more terms, he is able to calculate rather simply the rate of energy transformation into heat (and turbulence) in the jump from measured velocity distributions. This calculation which gives rates which are from 7 to 19% short of the actual values, shows that most of this transformation takes place between the high-velocity flow and the surface roller, but not along the bed.

H. A. Einstein, USA

714. G. I. Dvukhsherstov, Hydraulic shock in tubes of noncircular section and in the flow of a fluid between elastic walls (in Russian), Uchenie Zapiski Moskov. Gos. Univ. Mekh. 122, 17–76 (1948).

The purpose of this paper is to investigate the effect of the elasticity of the walls and the shape of the cross section upon the pressure behind, and the speed of propagation of the shock wave caused by the sudden closing of a valve in a pipe or conduit. The theory of shells is applied to determine the deformation of the cross section resulting from the hydraulic-shock pressures. Inertia of the walls is neglected. Shapes considered are rectangles, regular polygons and ovals of small noncircularity. The reduction of peak pressure resulting from even quite small noncircularity is very substantial. The effect of a short length of pipe of noncircular section placed between the valve and a long section of circular pipe is also discussed and apparently, when properly chosen, may also cause a substantial reduction.

J. V. Wehausen, USA

715. J. Smetana, Study of the surface of the tail flow of large barrages (in French), Rév. gen. Hyd. 14, 185–194 (July 1948); 15, 19–32 (Jan. 1949).

The paper is a comment on the Creager profile for the downstream face of a spillway dam. Where the length profile of the masonry of a spillway dam is identical with the profile of a free spilling nappe measured by Bazin (1898), then no negative pressures will be produced on the downstream face of the dam. The Creager profile for spillway dams is a profile giving always slightly positive pressures. When y is the vertical and z the horizontal axis and with $y = 0$ and $z = 0$ at the top of the dam crest, the Creager profile is given by $y = 0.47\, Z^{1.80}$ (Scimemi, 1937).

The author proposes now the curve $y = 0.461\, Z^{1.85}$, and gives his reasons for so doing. He calculates the weir-discharge coefficient for the Creager-Smetana profile on the basis of the well-known formula of Bazin for sharp-crested weirs.

Charles Jaeger, England

716. L. Escande, Studies on the simultaneous operation of movable dams in weirs and bottom outlets (in French), Houille Blanche no. spéc. B, 728–742 (1948).

The operation of a flat vertical gate that is simultaneously overflowed and underflowed has been studied on models of various scales. It has been ascertained that the discharge and the pressures on the gate have nearly the values obtainable from the superposition of the two separate movements, over and under the gate, and that, on the contrary, the distribution of pressures on the ground is very different.

The experiments have shown that under certain charges H and heights of the gate Δ, the movement is not steady but alternating. Destruction of the gate can be avoided by means of aeration of the back surface of the gate, or by taking $\Delta \geq 0.6\, H$.

Giulio de Marchi, Italy

717. Michinori Kurihara, On the critical tractive forces (in Japanese), Rep. Res. Inst. Fluid Engng., Kyûsyû Univ. 4, 1–26 (Sept. 1948).

It is well known that sand grains on the bed of a channel stream begin to move when the tractive force acting on the bed exceeds a certain critical value. Analyzing the existing experimental results, the author obtains a definite curve by plotting $\tau_0/\beta g k(\rho_1 - \rho)$ against $k(\tau_0/\rho)^{1/2}/\beta\nu$, where τ_0 is the critical tractive (shearing) stress, k and ρ_1 are mean diameter and density of the grain, ρ and ν are density and kinematic viscosity of the fluid, g is the acceleration due to gravity, and β is a parameter allowing for the nonuniformity of the grains. Although some scatter of the points occurs, a distinct minimum is found at about $k(\tau_0/\rho)^{1/2}/\beta\nu = 25$. The author interprets the occurrence of the minimum as a synchronizing phenomenon between the grain size and scale of turbulence. He estimates the force acting on the grain by taking into account the fluctuations of transverse pressure gradient, as well as longitudinal and transverse velocity components, and obtains a curve which agrees well with the experimental results.

Itiro Tani, Japan

Incompressible Flow: Laminar; Viscous

(See also Revs. 643, 713, 763, 774, 795)

718. G. C. McVittie, A systematic treatment of moving axes in hydrodynamics, Proc. Roy. Soc. A, 196, p. 285 (1948).

Transformation of the hydrodynamic equations to general curvilinear coordinates in motion is carried out in the four-dimensional space-time of special relativity. The Newtonian equations follow as a first approximation by making the velocity of light c very large. The equation for the rate of change of the vorticity-tensor is obtained by taking the curl of the equations of motion. The equation of continuity is shown to be independent of the motion of the coordinate system. The equation of heat transfer in a nonviscous fluid is generalized to the form in which differentiation with respect to t is replaced by that with respect to s and the Laplacian operator ∇^2 by the d'Alembertian $\Box$. The Newtonian approximation is again retrieved by letting c become infinite after the transformations of coordinates into moving systems are completed. The theory is applied to the motion of a gas on the surface of the rotating spherical earth, leading to Patterson's result; Sawyer's theory of tropical cyclones is also discussed.

P. Y. Chou, China

719. G. C. McVittie, Two-dimensional fluid motion referred to a network of orthogonal curves, Proc. Roy. Soc. A, 196, p. 301 (1948).

In certain problems of aerodynamics and of dynamical meteorology the motion of the fluid at each point of space is, in the main, parallel to some member of a one-parameter family of planes or surfaces. The general method developed in the foregoing paper is applied to this special problem, adopting a network of orthogonal curves on the surface as the coordinate system. The equations of motion and of continuity are expressed in terms of the geodesic curvatures of the orthogonal curves. As particular cases,

Meyer's aerodynamic equations for the network fixed in space drawn on parallel planes, and a formula for a large-scale gradient wind on the surface of the rotating spherical earth are derived.

P. Y. Chou, China

720. **Tatomir Angelitch, On the application of Pfaff's method in fluid dynamics** (in French, with Serbian summary), Acad. Serbe Sci. Publ. Inst. Math. 2, 211–222 (1948).

The author shows that the theory of Pfaffian systems can be used to derive the dynamical equation for viscous fluids under the unnecessary and in general incorrect assumption that the motion is barotropic.

C. A. Truesdell, USA

721. **Giovanni Cocchi, On the general equations of motion in liquid flow** (in Italian), Mem. Accad. Sci. Ist. Bologna Cl. Sci. Fis. 3, 207–213 (1947).

722. **P. P. Kufarev, Solution of the problem of the boundary of an oil field for a circle** (in Russian), Doklady Akad. Nauk SSSR 60, no. 8, 1333–1334 (June 1948).

Continuing the work of Polubarinova-Kochina and Falkovich [Prikl. Mat. Mekh. 9, no. 1, p. 79 (1945); 11, no. 6, p. 664 (1947)], and using the method of conformal mapping, the author supplies an exact solution of the problem (solved approximately in the first cited paper): The oil field is a circle $|z| < 1$, the well is at $z_0 = h$, $0 < h < 1$; determine the change of the oil-field boundary. Ed.

723. **I. N. Snedden and J. Fulton, The irrotational flow of a perfect fluid past two spheres,** Proc. Cambridge philos. Soc. 45, 81–87 (1949).

The authors treat the boundary-value problem of an irrotational flow past two spheres of an incompressible nonviscous fluid. This problem was considered previously in both electrostatics and hydrodynamics. Solutions had been obtained but all in cumbersome form. The object of this paper is to show that by the use of a general formula for the velocity potential due to Weiss [same source, 40, 259–261 (1944)] a simple closed expression may be obtained for the solution of the simplest boundary-value problem involving two spheres. It is assumed that the line of centers of the spheres is parallel to the undisturbed uniform flow.

A. Gelbart, USA

724. **Luigi Castoldi, A generalization of vortexes in perfect "nonhomogeneous" fluids subject to conservative mass forces** (in Italian), Pont. Accad. Sci. Acta 11, 207–217 (1947).

725. **J. L. Synge, On the motion of three vortices,** Canad. J. Math. 1, no. 3, 257–270 (1949).

The author obtains in a simple way Gröbli's equations of the motion of three vortexes which he studies in analytic form. He first considers the rather easy case of fixed configurations, then passes on to variable configurations, the study of which allows him to show certain interesting properties of singular points.

L. Escande, France

726. **I. Cârstoiu, On the motion of alternating vortexes extending indefinitely in one direction** (in French), Disquisit. Math. Phys. 6, 225–233 (1948).

The author studies the motion of the semi-infinite vortex street considered previously by Synge, assuming that the cylinder producing the vortex system suddenly disappears (which the author interprets physically as its stoppage). It is found that the system will not remain stationary, as expected. The author considers the stability at the initial configuration by displacing one vortex. It is found that one of the necessary conditions for stability is the original von Kármán condition, but the present system is always unstable.

C. C. Lin, USA

727. **Gabriel Viguier, Vortical distribution of a viscous incompressible two-dimensional flow** (in French, with Croatian summary), Hrvatsko Prir. Dr. Glasnik, Mat. Fiz. Astr. 3, 209–212 (1948).

728. **Gabriel Viguier, Flow of a viscous incompressible fluid in a thin inclined tube** (in French, with Croatian summary), Hrvatsko Prir. Dr. Glas. Mat. Fiz. Astr. 3, 202–208 (1948).

729. **Georges Bouligand, A case of entrainment of a viscous fluid** (in French), C. R. Acad. Sci. Paris 226, no. 22, 1776–1778 (1948). Review delayed.

730. **G. I. Taylor, Air resistance of a flat plate of very porous material,** Rep. Memo. aero. Res. Counc. Lond. no. 2236, 4 pp. (1944, publ. 1948).

The resistance offered to an air stream by a sheet of porous material is usually described by means of a nondimensional coefficient, k, defined as the ratio of the difference in pressure across the plate to the mean dynamic pressure across the plate. This coefficient approaches the drag coefficient as the material of the plate becomes more and more porous. In this paper an approximate formula is developed for very porous plates which is more accurate than the assumption $k = C_D$. Two independent derivations are given which lead to the same formula. Both are based on the assumption that the flow through the plate is uniform over its surface. A typographical error has been made; formula (15) should be identical with (6).

As a measure of the range of validity of the approximate formula, the lateral velocity component is calculated. This component is, in general, found to be small, from which fact the author concludes that the approximate formula is probably valid for values of k even as large as 4.

Bruno A. Boley, USA

731. **A. D. Young and P. R. Owen, A simplified theory for streamline bodies of revolution, and its application to the development of high-speed low-drag shapes,** Rep. Memo. aero. Res. Counc. Lond. no. 2071, 21 pp. (1943, publ. 1949).

The authors assume that the theory of incompressible and inviscid flow may be used to develop shapes of bodies of low drag at high speeds. A favorable velocity distribution is suggested and the corresponding shape of the body of revolution is determined. As Kaplan's computation of potential flow about elongated bodies of revolution [Nat. adv. Com. Aero. Tech. Rep. no. 516 (1935)] is rather lengthy, the authors try to simplify his method. They assume that the velocity distribution due to the body is the same along a meridian as that of a spheroid of the same maximum relative thickness. They also use the well-known approximate connection of the radius of the cross section with the source-and-sink distribution on the axis which replaces the body. These two assumptions allow a quick evaluation of the coefficients in the series which Kaplan used for computing the potential function.

The simplified method gives reasonable results up to 30% relative thickness when compared with those of Kaplan's original procedure. In the last chapter a very simple approximation is given for the distribution of transverse-force coefficient for an elongated body of revolution at small angle of yaw, and for the influence of the yaw on its velocity distribution.

Irmgard Flügge-Lotz, USA

732. A. R. Howell, A theory of arbitrary aerofoils in cascade, Phil. Mag. 39, 913–927 (Dec. 1948).

The problem of potential flow past cascades of airfoils has been solved for thin airfoils and flat plates, and recently for airfoils similar to the Joukowsky type. The purpose of this paper is to give a solution for arbitrary airfoils in cascade.

Four conformal transformations are used in order to reduce the flow past the airfoils in cascade to that of a known flow past a circle. The basis of the method is the fairly simple transformation $\xi_1 = \tanh Z_1$, where Z_1 is the plane of the airfoils in cascade. The ξ_1-plane then consists of a single airfoil. Two successive transformations of the Joukowsky type, $w = \xi + C^2/\xi$, then transform, after proper adjustment of new coordinate axes, the single airfoil into an approximate circle. For some cascades, such as those with large pitch/chord ratios, only one Joukowsky transformation is sufficient. The last step is to transform the approximate circle into an accurate circle by methods such as those of Theodorsen and von Kármán. Lift coefficients and pressure distributions are determined.

The computations are numerically worked out and compared with experimental results for the Goettingen profile 436. The theoretical results are in reasonable agreement with the experimental values.

The author states that the method has limitations for small pitch/chord ratios. Wilhelm Spannhake, USA

733. Kōzi Hirose, On the conformal transformation of a wing lattice composed of airfoils of arbitrary shape (in Japanese), Trans. Soc. mech. Engrs. Japan 14, no. 44, part 3, 22–27 (Oct. 1948).

This paper presents an approximate method of finding the analytic function which transforms a lattice of thin airfoils of arbitrary shape in the Z-plane into a unit circle in the ζ-plane in the form:

$$Z = \frac{\lambda}{2\pi}\left[e^{-i\beta}\log\frac{\kappa\zeta + (1+\epsilon)}{\kappa\zeta - (1+\epsilon)} + e^{i\beta}\log\frac{\zeta + \kappa(1+\epsilon)}{\zeta - \kappa(1+\epsilon)}\right] + C_0 + C_1\zeta^{-1} + C_2\zeta^{-2} + \ldots \quad (1)$$

The method proceeds along lines similar to the reviewer's method for a single airfoil [J. Soc. aero. Sci. Nippon 9, 865–875 (1942)]. Thus, noting that, when $\epsilon = 0$, $C_n = 0$, (1) gives the transformation of a lattice of line segments into a circle, ϵ and C_n may be considered to be small for a lattice of thin airfoils. On this basis, ϵ and C_n are determined to the order of $O(\epsilon)$ corresponding to any given wing lattice. Formulas for the lift and velocity distribution over the airfoil section are also given, but no numerical example is worked out. Isao Imai, Japan

734. Kōzi Hirose and Busuke Hudimoto, On the theory of wing lattices composed of airfoils of arbitrary shape (in Japanese), Trans. Soc. mech. Engrs. Japan 14, no. 47, part 3, 21–24 (Oct. 1948).

Aerodynamic characteristics of a lattice of thin, slightly cambered airfoils placed in a uniform flow of an incompressible fluid, at a small angle of attack, is investigated theoretically. First, a lattice of line segments representing the chords of given airfoils is transformed conformally into a unit circle. Next, an appropriate system of multiplets is sought for to be placed at the center of the circle so that the resultant fluid velocity may be tangential to the surface of the airfoils. The lift and moment coefficients, as well as the velocity distribution over the airfoil, are then given in terms of the geometrical factors of the given wing lattice.

Isao Imai, Japan

735. Tomoo Ishihara, Approximate theory of a lattice of thin airfoils (in Japanese), Trans. Soc. mech. Engrs. Japan 14, no. 47, part 3, 44–49 (Oct. 1948).

Glauert's thin airfoil theory, which consists essentially in replacing a given airfoil by a vortex sheet situated along its chord, is extended to the case of a wing lattice. Simple formulas for the lift coefficient and the position of the aerodynamic center are derived. They appear to be useful for a thin, slightly cambered wing lattice with large gap-to-chord ratio, at a small angle of attack. Comparisons with other theories and experiments are included. Isao Imai, Japan

736. P. Groen, Contribution to the theory of internal waves, Koninklijk Nederlands Met. Inst. de Bilt no. 125. Meded. Verhand. B, part II, no. 11, 23 pp. (1948).

The author considers an incompressible inviscid fluid of density continuously variable with depth and extending from $z = -\infty$ to $z = \infty$ vertically. The fluid is at rest in the unperturbed state and the internal waves are treated as small perturbations. When the law of variation of the specific volume $S(z)$ is taken as $S(z) = S_0 + \frac{1}{2}(\Delta S)\tanh(2z/b)$, where ΔS is the total variation of specific volume and b is regarded as a measure of the thickness of the transition layer, it is found that the wave-length $2\pi\lambda$ and the period $2\pi\tau$ are related by

$$(g\Delta S/bS_0)\tau^2 = n(n+1)(2\lambda/G)^2 + (2n+1)2\lambda/G + 1.$$

Here $n = 0, 1, 2, \ldots$ and represents the order of the mode of oscillation. Thus when $\lambda \to 0$ the period approaches a minimum value. The existence of this minimum value appears to be a general feature not restricted to the above law of density. In the case of a rotating earth the author finds the same relation as that given above to exist between τ and $\lambda(1 - 4\omega^2\tau^2)^{1/2}$, where ω is the vertical component of the angular velocity.

L. M. Milne-Thomson, England

737. Alexander Weinstein, On surface waves, Canad. J. Math. 1, no. 3, 271–278 (1949).

The author discusses the various methods which have been utilized in the linearized theory of surface waves. With the aid of the eigenvalue method he gives the complete solution of Airy's problem (plane waves in water of constant depth). Then he studies a problem treated by Stoker [Quart. appl. Math. 5, 1–54 (1947); Rev. 1, **179**]; this problem consists in the determination of waves in an ocean of infinite depth bounded on one side by a vertical cliff when the wave crests are not assumed to be parallel to the shore line. To solve this problem the author utilizes a combination of the eigenvalue method with the reduction method (choice of a new unknown function which satisfies the same partial differential equation as the first function, but for which the boundary condition takes a simplified form). The author obtains the complete solution of the problem. There is no solution which is regular everywhere. Ratip Berker, Turkey

738. Gerhard Neumann, On the tangential pressure of the wind and the roughness of the sea surface (in German), Z. Met. 2, 193–203 (July–Aug. 1948).

In this comprehensive and clearly written paper, the author discusses the theory of wave generation by wind passing over a water surface, and the interrelation between wind velocity, tangential stress and surface roughness. From observations of the slope produced by a steady strong wind sweeping over a water surface for an extended period of time, a relation between surface tangential stress (τ) and the wind velocity (v) is deduced. It is found that this relation differs from the quadratic relation τ =

$\rho' \kappa v^2$ found for wind sweeping over a rolling landscape (ρ' is the density of the air and κ is a coefficient of friction which varies with surface roughness).

The deviation from the quadratic law is explained by showing that the relation between κ and the wind velocity is approximately as $\kappa = 0.009v^{-1/2}$. From this it follows that the roughness of the water surface decreases with increasing wind velocity.

An attempt is made to explain this peculiar behavior of a rough water surface by the changing relation between wave length and wave height as the surface becomes rougher. The theoretical deductions agree well with experimental results. From this, new conclusions are drawn regarding the creation of waves by wind. Thus the creation of the first small wave appears to be accompanied by a sudden strong increase of the friction factor κ, which, however, tends to decrease again as the waves increase in height and length. Karl E. Schoenherr, USA

739. Sigeiti Moriguti, Water mamilla (in Japanese), Appl. Math. (Oyō-Sūgaku) 2, no. 1, 27–31 (1949).

The differential equation for the shape of a water droplet hanging down from a wet wall is derived by the calculus of variations from the condition that the sum of potential energy and surface energy (increase of surface area multiplied by surface tension) should be a minimum, subject to the condition of constant volume of the droplet, and to the obvious condition that the surface of the droplet should not be reentrant. At the beginning of the droplet formation, the square of the surface inclination may be neglected compared with unity, so that the differential equation may be integrated in terms of Bessel functions. The diameter of the droplet is determined from the eigenvalue of the equation. With usual values of physical constants, the diameter becomes about 2 cm, which agrees well with observation. Itiro Tani, Japan

Compressible Flow, Gas Dynamics

(**See also Revs. 731, 764, 769, 788**)

740. Robert Sauer, Relationship between the theory of deformation of surfaces and gas dynamics (in German), Arch. Math. 1, no. 4, 263–269 (1948/49).

Let u and v be Gaussian parameters on the surface $\mathbf{s} = \mathbf{s}(u,v)$. The infinitesimally deformed surface $\mathbf{s} + \epsilon\mathbf{t}$ is (in the small) isometric with s if $d\mathbf{s}.d\mathbf{s}$ includes no term linear in the infinitesimally small constant ϵ. It can be shown that in this case each surface element suffers a rotation $\epsilon\mathbf{r}$, where $\mathbf{r}$ satisfies the equations

$$\mathbf{r}_u = \mu\mathbf{s}_u + \lambda\mathbf{s}_v, \; \mathbf{r}_v = \nu\mathbf{s}_u - \mu\mathbf{s}_v \qquad (1a);$$

the functions λ, μ, ν must satisfy

$$(\mu\mathbf{s}_u + \lambda\mathbf{s}_v)_v = (\nu\mathbf{s}_u - \mu\mathbf{s}_v)_u \qquad (1b),$$

but are arbitrary otherwise. The surface $\mathbf{r}$ (the *Drehriss* of $\mathbf{s}$) is in point-to-point correspondence with $\mathbf{s}$.

If now u and v are interpreted as coordinates in the hodograph plane, then, for a given p, ρ-relation, the two-dimensional compressible flow equations are in the steady case equivalent to the system (1a,b) under the following correspondence: $\mathbf{s} \longleftrightarrow$ vector $(-v,u,p)$, $\mathbf{r} \longleftrightarrow$ vector (x,y,ψ), and $\lambda = -x_u$, $\mu = x_v = y_u$, $\nu = y_v$. Hence the *Drehriss* belonging to an infinitesimal deformation (in the above sense) of the pressure surface $p = f(u^2 + v^2)$ is a stream function $\psi(x,y)$ and conversely. A similar correspondence, viz. $\mathbf{s} \longleftrightarrow (-q,u,p)$, $\mathbf{r} \longleftrightarrow (x,t,\psi)$ etc., leads to the one-dimensional nonsteady flow equations, the independent variables being the velocity u and $q = -u^2/2 - \int dp/\rho$. In this case the pressure surface is $p = g(q + \frac{1}{2}u^2)$.

The first gas-dynamical case is thus mathematically equivalent to the infinitesimal deformation of a surface of revolution, the second to that of a parabolic surface of translation. Wherever the p-surface has negative curvature, there are real asymptotic lines on it which correspond to the hodograph characteristics; the reciprocal relation of these to the Mach lines in the physical plane appears here as a special case of a general relation between certain corresponding nets on the $\mathbf{s}$ and $\mathbf{r}$ surfaces. G. Kuerti, USA

741. K. P. Stanyukovich, The two-sided flow of a gas from a cylindrical vessel into a tube (in Russian), Doklady Akad. Nauk SSSR 58, 201–204 (1947).

Consider a cylindrical vessel at whose end is attached a tube. Let the gas in the cylinder be initially at a high pressure compared to that in the tubes, and let the ends of the cylinder be opened at different times. The author discusses the waves which arise, and their interpretation. G. F. Carrier, USA

742. W. P. Robbertse, On the compression and expansion phenomena in a gas caused by a collision of a piston moving with a velocity much exceeding that of sound (**Oor die Verdigtings-en Verdunningsverskynsels in 'n Gas Veroorsaak Deur die Stoot van 'n Suier met 'n Baie Hoë Snelheid**) (with English summary), Amsterdam: N. V. Noord-Hollandsche Uitgevers Maatschappij, 1948, vi + 104 pp.

Mostly very well-known material is presented, except for some approximate solutions satisfying exactly either only the equations or only the initial conditions. Ed.

743. D. C. Pack, The condition for the detachment of the shock wave from a wedge in a supersonic stream, Proc. Cambridge philos. Soc. 44, 298–300 (1948).

When a wedge is disposed symmetrically in a supersonic stream a shock wave is formed which is itself wedge-shaped and attached to the tip for sufficiently high Mach numbers but which, for lower Mach numbers, is curved and detached from the tip. Taylor and Maccoll in 1931 determined by graphical treatment of the shock equations the Mach number below which, for a given wedge angle, an attached shock configuration is impossible. The present author pursues the problem analytically and exhibits the critical shock-wave angle as a root of a quartic equation. D. P. Ling, USA

744. S. S. Byushgens, The critical surface of an adiabatic flow (in Russian), Doklady Akad. Nauk SSSR 58, 365–368 (1947).

The equation of continuity of a stationary adiabatic flow is cast into the form $(v^{-2} - a^{-2})dA/ds - \text{div}\, \mathbf{i} = 0$, where $v\mathbf{i}$ is the velocity vector, v the velocity, a the velocity of sound, s the arc length of a streamline, $a^2 = dp/d\rho$, $A = a^2/(k-)1$; density ρ and pressure p are related by $p/p_0 = (\rho/\rho_0)^k$. Moreover, Euler's equation gives $vdv/ds + dA/ds = 0$. Surfaces for which div $\mathbf{i} = 0$ are called minimal or critical surfaces; if div $\mathbf{i}$ is zero at all points, the congruence $\mathbf{i}$ is called minimal. Certain geometrical properties of of such congruences and surfaces are derived. For instance, all points where $v^2 = a^2$, $dA/ds \neq \infty$, lie on minimal surfaces. A minimal surface is the locus of points where either the velocity is equal to the velocity of sound or both these quantities have extreme values when changing along appropriate streamlines. In a plane adiabatic flow the velocity of the current can reach the velocity of sound only in the points of inflection of the orthogonal trajectories of the streamlines. Finally, necessary and sufficient condition that an adiabatic flow be critical ($v = a$) is that the congruence of streamlines be a special minimal congruence, that is, a

minimal congruence whose curvature vectors di/ds form a gradient field.

Courtesy of Mathematical Reviews D. J. Struik, USA

745. M. Schaefer, Formation of envelopes of curved Mach waves in flow along a convex wall, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1203-1A, 44 pp. (1949).

This is a continuation of a previous paper of the same author, which was restricted to the special case of a family of straight Mach waves emanating from the wall of a flow bounded on one side. For this case it was shown that the formation of envelopes could not occur without a curvature discontinuity in the wall.

In the present paper the fundamental question, whether the necessity of such a wall singularity might be removed by transition to the general case of a flow field in which both families of Mach waves are curved, is discussed and answered in the affirmative. In this case, the wall streamline bounding the flow is a smooth curve; however, the beginning of an envelope occurs at a place inside of the flow field.

In the new example, compression occurs on a convex wall, the curvature of which decreases considerably. The tendency of the Mach waves toward convergence caused by compression eventually overcomes the tendency of these waves toward divergence due to the curvature; finally, convergence wins the upper hand.

The paper refers to previous papers of F. Ringleb and W. Tollmien. Wilhelm Spannhake, USA

746. Jim Rogers Thompson, A rapid graphical method for computing the pressure distribution at supersonic speeds on a slender arbitrary body of revolution, Nat. adv. Comm. Aero. tech. Note no. 1768, 24 pp. (Jan. 1949).

This paper gives a graphical computing technique for the so-called "point-source method," developed from the results of Jones and Margolis, and based on a simplifying assumption concerning the source distribution used to represent the body. The validity of this assumption in the case of parabolic bodies is examined by comparison of the results by the point-source method with those obtained by the method of Jones and Margolis. The computational procedure is illustrated by an example.

Wilhelm Spannhake, USA

747. H. Reese Ivey and Robert R. Morrissette, An approximate determination of the lift of slender cylindrical bodies and wing-body combinations at very high supersonic speeds, Nat. adv. Comm. Aero. tech. Note no. 1740, 18 pp. (Oct. 1948).

Busemann's method of determination of forces at very high supersonic speeds, using the assumption $\gamma = 1.0$, is applied in this note to the inclined circular cylinder. The separation point on the cylinder is calculated on the assumption that it occurs at zero pressure coefficient, and the shape of the downstream shock is found by assuming no change in momentum after separation. The inaccuracies introduced by these somewhat drastic simplifications are not discussed. R. Smelt, USA

748. V. R. Thiruvenkatachar, The analogue of Blasius' formula in subsonic compressible flow, Proc. Nat. Inst. Sci. India 14, 339–342 (1948).

The following is an extract from the author's introduction. The object of this note is to derive a formula for the force in subsonic compressible flow, which is the analog of the well-known Blasius formulas in the incompressible case. The derivation is carried out on the basis of the hodograph method as recently developed by C. C. Lin. It is also known that the familiar Prandtl-Glauert rule is derivable from the formula. C. C. Lin, USA

749. Yuan Shen, The flow of a compressible fluid past quasi-elliptic cylinders at high subsonic speeds, Sci. Rep. Nat. Tsing Hua Univ. 5, 29–51 (1948).

The paper deals with the problem of constructing a solution for the flow of a compressible fluid past quasi-elliptic cylinders by the hodograph method. By starting with two power series representing the stream function of the incompressible flow past an elliptic cylinder, in the respective domains of convergence in the hodograph plane, the author constructs a corresponding solution for the compressible fluid by simply replacing the proper elementary solutions. The solutions thus obtained do not join to each other smoothly on the circle of convergence and consequently fail to represent the same flow. As an attempt to simplify the numerical calculation, the hypergeometric functions are approximated by $r(\tau)\ \{S(\tau)\}^n$ for the subsonic range, where τ is proportional to the square of the local flow speed. The forms of the functions $r(\tau)$ and $S(\tau)$ are not given explicitly, but it is indicated that they are determined numerically. Several numerical examples have been treated by this method, and the results are presented in tabular and graphical form.

In an appendix, the author makes brief reference to the work of Tsien and Kuo [Nat. adv. Comm. Aero. tech. Note no. 995 (1946)]. He points out that their method takes account of the continuity conditions at the circle of convergence, while his does not, and that there are other differences. Nevertheless, he believes that his numerical results are near to theirs.

W. R. Sears, USA

750. A. D. Young, Note on the limits to the local Mach number on an aerofoil in subsonic flow, Coll. Aero. Cranfield Rep. no. 14, 12 pp. (Apr. 1948).

Two discussions are presented indicating a rational approach to the explanation of the observed fact that local Mach numbers exceeding 1.4 are rarely found over airfoils in subsonic flows. The first proceeds from the observation that the pressure after the shock is not higher than the main-stream pressure. The second approach is based on the experimentally observed fact that the local Mach number around the surface is only a fractional part of Mach number calculated by means of the Prandtl-Meyer expansion; moreover, sonic flow first starts at a point somewhat behind the leading edge of airfoils at moderate angles of attack so that only a limited turning angle exists around the surface. In NACA RMA7B07 ("An empirically derived method for calculating pressure distributions over airfoils at supercritical Mach numbers and moderate angles of attack," by Gerald E. Nitzberg and Loma E. Sluder) the relation between the Prandtl-Meyer theory and the supercritical airfoil pressure distribution has also been investigated. It was pointed out that increase in Mach number is a function not only of the streamline deflection angle but also of the difference between the free-stream and the critical Mach number of the wing. Subsequent work ("Some fundamental similarities between boundary-layer flow at transonic and low speeds," by Gerald E. Nitzberg and Stewart Crandall; see Rev. 1, 992) has indicated that the limiting value of the Mach number ahead of a shock in supercritical flow may possibly depend upon considerations of the stability of the turbulent boundary layer.

Max A. Heaslet, USA

751. Stefan Bergman, Two-dimensional transonic flow patterns, Amer. J. Math. 70, 856–891 (Oct. 1948) = Harvard Univ. Grad. School Engng. NOrd 8555, Task F, tech. Rep. no. 10, 47 pp. (1948).

The author develops a general method for obtaining solutions of a linear partial differential equation in two variables. In the case of elliptic type, he introduces the so-called integral operator

of the first kind yielding the solution in terms of an arbitrary function of one complex variable. It is capable of representing the solution with assigned singular characters at given points. Analogous representations of solutions for the hyperbolic type are obtained in terms of two differentiable functions of one real variable. In applications to the hodograph equation of a two-dimensional compressible flow, he further introduces the so-called integral operator of the second kind in order to study the behavior of the flow near the sonic line. In the case of Tricomi's equation, the solutions in the subsonic and the supersonic regions are connected by analytic continuation.

The general question of combining the various representations of the solution in various domains into one by means of analytic continuation, as pointed out by the author, can be treated by a method representing a generalization of Fuchs's theory. This idea will be developed in a subsequent paper by the author.

The alternative version gives additionally the complete calculations leading to the results stated. S. S. Shu, USA

752. F. I. Frankl', On a family of particular solutions of the equation of Darboux-Tricomi and their application to the approximate calculation of the critical current in a given plane-parallel Laval nozzle (in Russian), Doklady Akad. Nauk SSSR 56, 683–686 (1947).

In a previous paper [Bull. Acad. Sci. URSS Sér. Math. 9, 387–422 (1945)] the author showed that in the neighborhood of the sonic line the motion of a gas in a two-dimensional Laval nozzle may be described approximately by the Darboux-Tricomi equation (*) $\eta\psi_{\theta\theta} + \psi_{\eta\eta} = 0$. Here η is a certain function of the speed, θ the inclination of the velocity vector, and $\psi = \psi(\eta, \theta)$ the stream function. In this paper the following solutions of (*) valid in both the elliptic and hyperbolic half-planes are discussed:

$$\psi_n = (\mu - \lambda)^{\alpha_n} F[-\alpha_n, \tfrac{2}{3} - \alpha_n, \tfrac{5}{6} - \alpha_n, \mu/(\mu - \lambda)],\ n = 0, 1, \ldots,$$

where μ, λ are the characteristic coordinates:

$$\mu, \lambda = \theta \pm \tfrac{2}{3}(-\eta)^{3/2},\ \alpha_n = (2n + 1)/3,$$

and F denotes the hypergeometric function. Various formulas for ψ_n are given and it is shown that these functions are algebraic. [Tricomi already showed that for $n \equiv \pm 1 \pmod 3$ ψ_n is a polynomial in θ, η.] For $n \equiv 0 \pmod 3$ and $\lambda < 0 < \mu$, ψ_n has three real branches. The author announces another particular solution Ψ_t of (*) depending upon a real parameter t which was found by Falkovich. The formula for Ψ_t is too complicated to be reproduced here. A finite sum of the form $\psi = A\,\Psi_t + \sum B_n\psi_n$ represents a transonic flow through some Laval nozzle L. An approximate procedure for finding the flow through a given nozzle $\bar{L}$ slightly different from L is described briefly. L. Bers, USA

753. K. O. Friedrichs, On the non-occurrence of a limiting line in transonic flow, Commun. appl. Math. 1, 287–301 (Sept. 1948).

With the free-stream Mach number M less than one, the flow over a body can be supersonic in some limited region adjacent to the profile. The author assumes that the space coordinates (x, y) are analytic functions of the velocity components (u,v), satisfying the appropriate differential equations and the boundary condition that the image of the boundary of the domain in the (u,v) plane (in which the pair (x,y) is defined) lies on a prescribed contour in the (x,y) plane with a finite curvature. The author claims that he has proved the falsehood of the following statement: There is a set of the coordinate functions depending continuously on M, such that the Jacobian $J = \partial(x,y)/\partial(u,v)$ does not vanish for $M < M_0$, while $J = 0$ somewhere for $M \geq M_0$. The proof consists of two parts: (1) $J \neq 0$ at a boundary point for $M = M_0$; (2) if $J = 0$ at some interior point, then $J < 0$ somewhere. The reviewer has difficulty in accepting the second as part of the proof for nonoccurrence of a limiting line (line where $J = 0$). This is because of the fact that when the limiting line occurs, the flow in the physical plane has folds. One branch of solution in the folded region has negative values of J. Thus it seems to the reviewer that the author simply has demonstrated the trivial fact that in a flow where no limiting line is allowed, there is no limiting line. H. S. Tsien, USA

Discussion on foregoing review:

Since the main point of my indirect argument was misunderstood, I should like to summarize its logic. Suppose there were a Mach number for which the Jacobian J of the solution of the hodograph equations vanished somewhere, then there would be a smallest such Mach number M_0, under the circumstances considered. Since $J > 0$ everywhere for $M < M_0$, evidently $J \geq 0$ everywhere, but $J = 0$ somewhere for $M = M_0$. The theorem formulated at the end of page 290 excludes just this possibility: $J \geq 0$ everywhere and $J = 0$ somewhere, since $J = 0$ at the walls was excluded by other arguments. Accordingly, a transition Mach number M_0 does not exist and, hence, J does not vanish for any Mach number. In other words, no limiting line does occur.

The difficulty in accepting this proof is apparently due to the failure to realize that the indirect argument is used only for the transition Mach number M_0 for which the condition $J \geq 0$ is satisfied automatically and not imposed arbitrarily; for which, in other words, a full-fledged limiting line has not yet developed.

If it were true, as it seems to the reviewer, that I had simply demonstrated that no limiting line exists where none is allowed, the argument could indeed have been simplified by reference to Ch. Morgenstern. K. O. Friedrichs, USA

In the author's own words in his comment on the review, his investigation is intended for cases where "a full-fledged limiting line has not yet developed." Therefore the author has definitely proved the following statement: For cases where a full-fledged limiting line has not yet developed, there is no limiting line.

The question of whether one can derive satisfaction by knowing the existence of a proof of the above statement has to be decided by one's taste. The reviewer's remark in the original review was really prompted by the thought that by assuming the existence of the solution of the desired type one automatically rejected the limiting line. The appearance of the limiting line is only a symptom. The deeper problem is the existence theorem of compressible flow over a specified smooth body. The existence proof is of course very difficult. H. S. Tsien

I disagree with the reviewer's interpretation of what I have proved, but I perfectly agree with his last three sentences.

K. O. Friedrichs

754. Walter Wuest, On the theory of the bifurcated compression shock (in German), Z. angew. Math. Mech. 28, 73–80 (Mar. 1948).

A simplified graphical method of computing bifurcated compression shocks is given. In certain limiting cases, where one of the shocks degenerates into a Mach wave, the method fails. These cases are treated analytically and the equations governing these limiting cases given. The region in which solutions leading to bifurcated shock is shown by means of various diagrams.

J. A. Lewis, USA

755. Wilber B. Huston, Calvin N. Warfield and Anna Z. Stone, A study of skin temperatures of conical bodies in supersonic flight, Nat. adv. Comm. Aero. tech. Note no. 1724, 42 pp. (Oct. 1948).

An empirical expression due to Eber (Archiv Nr. 66/57, Peenemünde, November 21, 1941) for heat-transfer coefficients on conical bodies in supersonic flow is found to give moderate agreement with the measured skin-temperature history during one V-2 missile flight. Encouraged by this agreement, the authors have computed skin temperatures for a wide range of pertinent parameters. Stanley Corrsin, USA

756. A. Ya. Sagomonyan, Operational methods in gas dynamics (in Russian), Vestnik Moskov. Univ. **3**, no. 5, 53–58 (1948).

The operational calculus is incorrectly applied to the problem of an oscillating deformable obstacle in a supersonic flow.
G. F. Carrier, USA

757. I. E. Garrick and S. I. Rubinow, Theoretical study of air forces on an oscillating or steady thin wing in a supersonic main stream, Nat. adv. Comm. Aero. Rep. no. 872, 14 pp. (1947, publ. 1949).

In a supersonic flow of Mach number M in the X-direction, if an oscillating thin wing is placed on the XZ-plane, then according to the linearized theory, the boundary condition is specified by the normal (y-component) disturbance velocity $w(x, z; t)$ at the XZ-plane. Let $w(x, z; t) = W(x, z)w(t)$. The authors show that the disturbance velocity potential ϕ in the XZ-plane is given by

$$\phi(x, z; t) = -\frac{1}{2\pi\beta}\int_0^x\int_{\zeta_1}^{\zeta_2} W(\xi, \zeta) \times \frac{w(t-\tau_1) + w(t-\tau_2)}{\sqrt{(\zeta-\zeta_1)(\zeta_2-\zeta)}}\, d\zeta d\xi,$$

$$\tau_1 = \frac{M(x-\xi)}{c\beta^2} - \frac{\sqrt{(\zeta-\zeta_1)(\zeta_2-\zeta)}}{c\beta},$$

$$\tau_2 = \frac{M(x-\xi)}{c\beta^2} + \frac{\sqrt{(\zeta-\zeta_1)(\zeta_2-\zeta)}}{c\beta},$$

$$\zeta_1 = z - \zeta_0,\ \zeta_2 = z + \zeta_0,\ \zeta_0 = \frac{x-\xi}{\beta},$$

$$\beta = (M^2-1)^{1/2},\ c \text{ sound velocity},$$

and $W(\xi, \zeta) = 0$ at points where the integrand is not real.

For purely supersonic cases, all necessary information on $W(x, z)$ can be deduced from the specified geometry of the wing. The calculation is then straightforward. The authors indicated the results for these cases: wings of infinite span with or without sweep, delta wings with supersonic leading edges. The detailed results are given in another paper [same source, tech. Note no. 1383 (1947)]. For other cases such as rectangular wings, the appearance of effectively subsonic regions makes the specification of $W(x, z)$ in these regions difficult. The authors were not able to give definite answers to these cases. However Evvard [same source, tech. Note no. 1699 (1948)] has shown how this difficulty can be overcome for a wide class of problems. H. S. Tsien, USA

758. Harold Grad, Resonance burning in rocket motors, Commun. appl. Math. **2**, 79–102 (Mar. 1949).

Resonance burning is the phenomenon of resonance between the oscillations of gas pressure (during the burning of hollow powder grains) and the burning rate of the powder. This resonance produces abnormally sharp pressure rises in the combustion chamber, as well as pitting and ripples on the inside surface of the grains. The author's explanation assumes the dependence of the burning rate of the powder on pressure and temperature. For a qualitative explanation, the author studies the natural modes of oscillation of a gas flow inside a cylindrical channel closed at one end, the lateral surface of which is crossed inward by a mass-flow rate equal to the mass of gas generated in unit time owing to the combustion: the equations of motion are linearized and the boundary condition which the flow must fulfill are determined. The author deduces the conditions under which the fluctuations are cumulative; they involve the dependence of the burning rate on the pressure and temperature, and the Mach number of the gas flow at the inner surface of the powder grain. He finally determines the mode of oscillation which builds up the most rapidly. Carlo Ferrari, Italy

759. S. Corrsin and M. S. Uberoi, Further experiments on the flow and heat transfer in a heated turbulent air jet, Nat. adv. Comm. Aero. tech. Note no. 1865, 61 pp. (Apr. 1949). Supplement to Rev. **3**, **136**.

The effective turbulent Prandtl number for a section of the fully developed jet was found to be equal to the true (laminar) Prandtl number within the accuracy of measurement.

Measurements of turbulence level (u', v'), temperature fluctuation level ϑ', and temperature-velocity correlation $\overline{\vartheta u}$ permit a comparison of their relative magnitudes.

Direct measurements have been made of the double correlations $\overline{uv}$ and $\overline{\vartheta u}$ across a section of the fully developed jet, and the shear-stress and heat-transfer distributions have been computed therefrom. Finally, these last-mentioned measurements have permitted a determination of the distribution of turbulent Prandtl number across the jet, and these values agree quite well on the average with the effective value computed from mean velocity and temperature alone. Authors' summary

760. Michel Luntz, Molecular aerodynamics (in French), Rech. aéro. Paris no. 8, 9–12 (Mar.–Apr. 1949).

Continuing his investigation of free molecular flows [same source, no. 7, 17–33 (1949); see Rev. **2**, **1040**], the author calculates the temperature and the polar of a polished-aluminum flat wing with radiation equilibrium. The solar radiation is not considered, but the earth's radiation is taken into account by assuming a constant earth temperature of 15 C. Two types of wing are considered: thermal insulation between the top and the bottom surfaces, and perfect conductivity between these surfaces. The atmosphere is assumed to have the same characteristics as that at ground. Results are presented in graphs for two Mach numbers 2.39 and 9.55. H. S. Tsien, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 713, 717, 759)

761. Gwoh-Fan Djang, Solution of Prandtl's boundary layer equation by a modified iteration method, Acad. Sinica Science Record **2**, 155–157 (1948).

This note gives an iteration process for the boundary-layer equation by calculating the shear from the velocity by the momentum relation and the velocity from shear by Newton's law. A satisfactory result is obtained for the Blasius case.
C. C. Lin, USA

762. Hikoji Yamada, An approximate method of integration of laminar boundary-layer equations, I (in Japanese), Rep. Res. Inst. Fluid Engng. Kyûsyû Univ. **4**, 27–42 (Sept. 1948).

An attempt is made to refine Pohlhausen's approximate solu-

tion by taking a polynomial of degree six in y as the velocity profile and satisfying, in addition to the usual momentum equation, the integral relations obtained by multiplying the equations of motion by y and y^2 respectively, and then integrating across the boundary layer. The calculation is reduced to the solution of simultaneous differential equations for the functions $\lambda(x)$, $a_5(x)$ and $a_6(x)$, where λ is the Pohlhausen parameter, and a_5 and a_6 are the coefficients of y^5 and y^6 in the polynomial respectively. In this preliminary report, the second integral relation is abandoned, and a_5 or a_6 is simply put equal to zero. The method, when applied to the flows for which exact solutions are known, results in promising approximations. Itiro Tani, Japan

763. Robert T. Jones, Effects of sweep-back on boundary layer and separation, Nat. adv. Comm. Aero. Rep. no. 884, 3 pp. (1947, publ. 1949).

For an infinitely long cylinder in an oblique viscous flow, the flow pattern and pressure disturbances at right angles to the axis of the cylinder are determined solely by the component of velocity in these planes. This fact enables the author to describe boundary-layer behavior on sweptback wings of high aspect ratio. He discusses very briefly boundary-layer thickness, stability, separation and the maximum lift.

I. Flügge-Lotz, USA

764. Edmond Brun and Marcel Vasseur, Some definitions concerning the boundary layer; A contribution to the thermal study of the laminar boundary layer (in French), J. Rech. Centre Nat. Rech. Sci. 2: 118–120, 121–126 (1948).

The first of these two papers presents definitions and physical interpretations of several characteristic lengths defined in terms of the density, velocity and temperature distributions of the boundary layer. Some of these characteristic lengths arise in the second paper which deals with the two-dimensional boundary layer in a heat-conducting compressible fluid where the fluid density ρ, the viscosity μ, and the thermal conductivity λ may be considered functions of the temperature only. Consequently the analysis holds only for moderate fluid velocities where the variation of density with pressure may be neglected.

The authors propose an iteration solution such that in the first approximation ρ, μ, λ are considered constant, and in the nth approximation the density, viscosity, and thermal conductivity distributions used would be those calculated from the temperature of the $(n-1)$th approximation. The analysis is restricted to the first approximation inasmuch as the thermal properties are of primary interest. The first approximation velocity distribution is independent of the temperature distribution and is identical with that of the boundary layer in an incompressible fluid. The temperature distribution is then described by the energy equation using the known velocity components for the incompressible boundary layer. When the free stream velocity distribution is prescribed to be $U \sim x^m$, $m = \beta/(2-\beta)$, where β is arbitrary and x is the distance along the surface measured from the forward stagnation point, then the differential equation for the temperature distribution $\theta(n, x)$ is

$$xf'\frac{\partial\theta}{\partial x} - \frac{f}{2-\beta}\frac{\partial\theta}{\partial\eta} - \frac{\lambda}{(2-\beta)\mu c_p}\frac{\partial^2\vartheta}{\partial\eta^2} = \frac{U^2}{c_p(2-\beta)}(f''^2 - \beta f'),$$

where c_p is the specific heat of the fluid at constant pressure, η is the dimensionless distance measured normal to the surface:

$$\eta = y\left\{\tfrac{1}{2}(m+1)\rho U/\mu x\right\}^{1/2},$$

and $f(\eta)$ is the dimensionless form of the stream function of the incompressible boundary layer. The values of $f(\eta)$ have been determined by Hartree [Proc. Cambridge philos. Soc. 33, 223–239 (1937)] for $-0.198 \leqq \beta \leqq 1.60$. By calling

$$\vartheta(\eta, x) = x^{2m}\theta_1(\eta) + \theta_2(\eta, x),$$

the ordinary differential equation

$$2\beta f'\theta_1 - f\theta_1' - \frac{\lambda}{c_p\mu}\theta_1'' = \frac{U^2}{2c_p}(2f''^2 - 2\beta f'),$$

and the homogeneous partial differential equation

$$(2-\beta)xf'\partial\theta_2/\partial x - f\partial\theta_2/\partial\eta - (\lambda/c_p\mu)\partial^2\theta_2/\partial\eta^2 = 0$$

are to be satisfied.

The authors consider the problem of (1) determining the temperature distribution of the wall so that no heat is transferred from the boundary layer, and (2) finding the rate of heat transfer when the wall temperature is prescribed. Each of these problems is reduced to the numerical integration of an ordinary differential equation. The authors discuss briefly the possible solution of problems where the free stream velocity is prescribed arbitrarily.

Courtesy of Mathematical Reviews F. H. Marble, USA

765. Kazunori Kitajima, On the mixing length of turbulence (in Japanese), Rep. Res. Inst. Fluid Engng. Kyûsyû Univ. 4, 43–54 (Sept. 1948).

For the isotropic turbulence of medium wave numbers, the "law of five-thirds" $q_n/n = (\epsilon/A)^{2/3}n^{-5/3}$ has been known for the spectral function, where $q_n dp_n$ is the proportion of u^2 due to components comprised in the interval dp_n, $n = \exp p_n$ the wave number, ϵ the volume rate of total dissipation, and A a nondimensional constant (Obukhoff, 1941). With the help of the experimental evidence that the spectral function preserves its shape during the decay in the case of turbulence in the wind stream behind a grid, the author assumes that the rate of decay for the eddies of wave number k would be

$$\epsilon q_k/{}_n\int^\infty q_k dp_k = (2\epsilon/3)(n/k)^{2/3},$$

if there were no energy supply from the eddies of wave numbers smaller than n. Writing the transition function for energy between the intervals $dp_i (i < n)$ and dp_k $(k > n)$ in the form

$$f(i, k)dp_i dp_k,$$

the author puts ${}_0\int^n f(i,k)dp_i$ equal to $(2\epsilon/3)(n/k)^{2/3}$, whence he obtains the result $f(i, k) = (4/9)Ai\, q_k\sqrt{q_i}$.

In the case of flow in pipes or channels, the author takes the view that the eddies of medium wave numbers are isotropic and governed by a definite law of statistical equilibrium, irrespective of the nature of mean flow. The decay of energy in this part is now to be compensated by the vorticity diffusion due to large eddies of nonisotropic character. The supposition leads the author to formulate the equations of transference of energy and vorticity for the flow in pipes or channels. Itiro Tani, Japan

Aerodynamics of Flight; Wind Forces

(See also Revs. 634, 732, 750)

766. John B. Parkinson, Roland E. Olson and Marvin I. Haar, Tank investigation of a powered dynamic model of a large long-range flying boat, Nat. adv. Comm. Aero. Rep. no. 870, 23 pp. (1947, publ. 1949).

Principles for designing the optimum hull for a large long-range flying boat to meet the requirements of seaworthiness, minimum drag, and ability to take off and land at all operational gross loads were incorporated in a $^1/_{12}$-size powered dynamic model of a four-engine transport flying boat having a design gross load of 165,000

lb. These design principles included the selection of a moderate beam loading, ample forebody length, sufficient depth of step, and close adherence to the form of a streamline body.

The aerodynamic and hydrodynamic characteristics of the model were investigated in Langley tank no. 1. Tests were made to determine the minimum allowable depth of step for adequate landing stability, the suitability of the fore-and-aft location of the step, the take-off performance, the spray characteristics and the effects of simple spray-control devices. The test results indicated that: Landing stability was satisfactory with a depth of step of 9% beam at the centroid; the hydrodynamic center-of-gravity range for stable take-offs was satisfactory as to extent and position with respect to the stable flight range desired; the take-off performance was satisfactory for the power loading assumed; the relation of the proportions to the design loading of the hull was correct for satisfactory spray characteristics; and large overloads were possible with relatively simple spray-control devices. The application of the design criterions used and of the test results should be useful in the preliminary design of similar large flying boats. Ernest G. Stout, USA

Propellers, Fans, Turbines, Pumps, etc.

767. Duilio Citrini, A theory of the hydraulic injector (in Italian), Energia Elett. 25, no. 12, 621–637 (Dec. 1948).

The analysis of the motion in the mixing chamber of a hydraulic injector (jet pump) is developed for incompressible fluids, without using the usual, but incorrect, assumptions of constant pressure head and negligible loss of head. The increase in the pressure head along the device is evaluated. For the loss of head an approximate analytical expression is given. The efficiencies of the device for different situations are examined, and examples of practical calculation are given.

The theory is developed for the injector with a cylindrical mixing chamber, but the case of a convergent mixing chamber is also examined, according to a design proposed by Rateau; the comparison shows that the cylindrical type provides, in general, a higher efficiency. Giulio de Marchi, Italy

Flow and Flight Test Techniques

768. Arthur F. Scott, David P. Shoemaker, K. Nolen Tanner, and James G. Wendel, Study of the Berthelot method for determining the tensile strength of a liquid, J. chem. Phys. 16, 495–502 (May 1948).

It is found that the high values of hydrostatic tension found by other investigators using Berthelot tubes must be ascribed to the presence of a trace of air in the tube. A method is described for preparing Berthelot tubes in an air-evacuated system, using water from which air has been carefully removed. The average value of the maximum hydrostatic tension obtained using these especially prepared Berthelot tubes was 32 atm. This is in excellent agreement with the value obtained by Meyer (34 atm) who used a method which avoided the doubtful assumption of the Berthelot method. R. L. Bisplinghoff, USA

769. J. A. Beavan and A. R. Manwell, Tables for use in the determination of profile drag at high speeds by the pitot traverse method, Rep. Memo. aero. Res. Counc. Lond. no. 2233, 8 pp. and 17 pp. of tables (1941, publ. 1949).

Let H denote the total head, p the (static) pressure; let the subscript 0 refer to an undisturbed upstream section and 1 to the traversed downstream section, then the drag coefficient in supersonic compressible flow is obtained by evaluating the well-known integral

$$(1)\quad \tfrac{1}{2} C_D = \int (H_1/H_0)^{\beta}(p_1/p_0)^{1/\gamma}(a_1/a_0)^{1/2} \times \left\{1 - [1-(b/a_0)]^{1/2}\right\} d(y/c),$$

where $a_i = 1 - (p_i/H_i)^{\beta}$, $b = 1 - (p_0/H_1)^{\beta}$, and $\beta = (\gamma - 1)/\gamma$.

In this paper the integrand in (1) is cast, for $\gamma = 1.40$, in the form

$$(2)\quad \tfrac{1}{7} Qh + \epsilon_1 + \epsilon_2 s + \epsilon_3 s^2,$$

where $Q^{-1} = -a_0$, $-h = (H_1/H_0) - 1$, and $s = (p_1/p_0) - 1$.

The ϵ_i are functions of h and z, Q depends on z alone, and

$$z = (H_0/p_0) - 1.$$

The functions $Q(z)$ and $\epsilon_i(h, z)$ have been tabulated, the latter three with entries ϵ and z. The form (2) of the integrand together with the tabulated functions is particularly useful if the integration is performed by the trapezoidal rule. G. Kuerti, USA

770. Pietro Teofilato, Derivation of wind-tunnel results from those in a water tunnel (in Italian), Pont. Acad. Sci. Acta 11, 109–116 (1947).

771. L. I. G. Kovasznay, Some improvements in the hot-wire anemometry (in English), Hung. Acta Phys. 1, no. 3, 25–51 (1948).

The general theory of the hot-wire anemometer is reconsidered in this article, including the effect of the changes in resistance of the wire on the heating current. The heating-current changes reduce the rate of change in resistance with air speed, even in the absence of thermal lag, and thereby cause considerable losses in sensitivity unless high-voltage heating circuits are used. The time constant characterizing the response of the hot wire to air-speed fluctuations is also reduced. A new method of compensation of the wire response for thermal lag is also presented by the author. It consists of a two-stage amplifier having a direct stage of uniform response and a differentiating stage with a response linear with frequency and with a phase lag independent of frequency. Changes of the amplification of the differentiating stage control the value of the time constant. The proper value of this time constant is determined in a bridge circuit having a heating current fluctuating between two constant values. The distortion of this square-wave current fluctuation caused by the thermal lag appears on an oscilloscope screen and the proper amount of compensation can be determined visually with great accuracy. Andrew Fejer, USA

772. W. A. Mohun, Precision of heat transfer measurements with thermocouples—insulation error, Canad. J. Res. Sec. F, 26, 565–583 (Dec. 1948).

A simple method has been developed for calculating the reading error of an insulated thermocouple due to conduction along the lead wires. The difference between the temperature of the junction and that of the surrounding material that it purports to measure is shown to depend on the temperature of the path, followed by the lead wires only within a distance from the junction called the "critical distance" s_1. In terms of the thermal conductivity of the wire K, its diameter D, the thermal conductivity of the insulation k, and its thickness w: $s_1 = (wDK/2k)^{1/2}$. The excess temperature of the junction over the temperature of its immediate surroundings, T_0 is equal to $\frac{1}{2}(T_{s_1} - T_0)$, where T_{s_1} is the temperature of the material around the leads at a distance from the junction equal to s_1. Detailed calculations are presented for enameled thermocouples located in a chordal hole in a 1-in. steel pipe.

To eliminate reading errors for insulated thermocouples, the

path of the leads needs to be isothermal only for the critical distance. When the path of the wires cannot be made isothermal, the conditions for minimum experimental error are shown to be small-diameter wires of low thermal conductivity with a minimum of insulation. R. L. Pigford, USA

Thermodynamics

(See also Revs. 758, 789)

773. J. E. Verschaffelt, The laws of reciprocity and superposition in thermomechanics (in French), Bull. Acad. Belg. Cl. Sci. 34, no. 2, 126–151 (1948).

A principle of superposition relative to the independent action of elementary irreversible processes is proposed. Closely related is the concept of fields of affinity. There is some question whether these propositions are consistent with other established theories of the thermodynamic behavior of gases [I. Prigogine, Rev. 2, **1065** (1949)] and the theories of Chapman and Cowling.

Newman A. Hall, USA

774. J. E. Verschaffelt, On the diffusion of gases (in French), Bull. acad. Belg. Cl. Sci. 34, no. 6, 500–517 (1948).

The question of mean diffusion velocities for multicomponent systems is considered relative to the action of concentration, temperature and pressure gradients. The method of the preceding review is used, and conclusions are uncertain to the extent of the limitations of that theory. Newman A. Hall, USA

775. S. R. de Groot, On the thermodynamics of certain irreversible processes, I. Simple bodies, II. Thermal diffusion and related phenomena (in French), J. Phys. Radium 8, 188–191, 193–200 (1947).

Starting from the "flux-force" relations $J_i = \Sigma_k L_{ik} X_k$ and Onsager's relations $L_{ik} = L_{ki}$, the author develops the known formula $\Delta p/\Delta T = -Q^*/vT$ for a bipartite adiabatically isolated body, where $Q^* = Q - w$, $Q = L_{12}/L_{11}$, and $w = \mu + Ts$.

For a mixture of j components, the author represents the flux of matter by $J_i = \Sigma_k a_{ik} X_k + b_i X_u$ $(i = 1, \ldots, j)$ and the flux of energy by $J_u = \Sigma_k b_k' X_k + c X_u$. Using Onsager's relations

$$a_{ik} = a_{ki} \text{ and } b_i = b_i',$$

the author develops the theory of the Soret effect and related effects. C. C. Torrance, USA

Courtesy of Mathematical Reviews

776. J. A. Prins, A thermodynamical substitution group, Physica 13, 417–421 (1947).

The author gives a geometric diagram for the group of substitutions among P, V, T, S, and U, W, F, G, leaving the thermodynamic equations invariant. Ed.

777. O. K. Rice, The effect of pressure on surface tension, J. chem. Phys. 15, 333–335 (May 1947).

This article constitutes an interpretation of the Lewis and Randall equation for surface-tension behavior in a multicomponent, two-phase system,

$$(\partial\gamma/\partial p)_{\sigma T} = (\partial V/\partial\sigma)_{pT},$$

where γ is the surface tension, σ is the interfacial area, V is the total volume of the system, and p is the total pressure.

For an inert gas over a liquid, the term $(\partial V/\partial\sigma)_{pT}$ is conceived to arise from two sources: (1) the volume $\Gamma kT/p$ (for an ideal gas) removed from the gaseous phase by adsorption of the gas at the interface. Here Γ denotes the number of molecules per unit area of surface region in excess of what would exist if the interior-liquid-gas concentration persisted precisely to the interface, and k is Boltzmann's constant; (2) the volume change ΔV_σ caused by an increase of surface region having a density unequal to that of the liquid interior.

The author sets ΔV_σ equal to zero and then uses experimental values of $(\partial\gamma/\partial p)_{\sigma T}$ to make order-of-magnitude estimates of Γ. He arrives at numbers approximating $\Gamma = 5 \times 10^{13}$ molecules/cm^2 and concludes that "in no case is a completely monomolecular layer very closely approached." Harold G. Elrod, Jr., USA

778. Richard C. Tolman, The superficial density of matter at a liquid-vapor boundary, J. chem. Phys. 17, 118–127 (Feb. 1949).

The author calculates the distribution of matter within the transition layer between the vapor and liquid phases of a single component fluid system on the following basis. The van der Waals equation of state is assumed to hold in both phases, the values of the two parameters in the equation being determined by the ratio of the bulk densities of the two phases. It is found qualitatively that on passage through the transition layer the density rises continuously from the value in the bulk vapor to a maximum value possible for the vapor, then rises abruptly to a minimum value possible for the liquid and finally rises again continuously to the value which it has in the bulk of the liquid. A corresponding trend is found for the pressure. To obtain quantitative results it is necessary to approximate the continuous trends of density or pressure by means of an analytic function of position. These functions for the pressure are then introduced into previously derived expressions [same author and source, 16, p. 758 (1948)] for the magnitude and location of the surface of tension in order to obtain an expression for Γ, the superficial density of matter, as defined by the Gibbs adsorption isotherm and referred to the surface of tension. This expression is in terms of the surface tension, the molecular weight, the temperature and a calculated function of the ratio of the densities in the two bulk phases. Results are tabulated for water, methyl and ethyl alcohol, acetic acid and ethyl ether at 20 C, and for water for the temperature range from 10 to 50 C. Γ is found to be positive in all cases and of the order of 0.8 to 2.5×10^{-8} g/cm^2. The point of abrupt rise of density does not occur at the surface tension but 0.3 to 0.7 of an intermolecular distance to the vapor side of it. The validity of the various assumptions is discussed.

John T. Burwell, Jr., USA

779. M. E. Fine and W. C. Ellis, Thermal expansion properties of iron-cobalt alloys, Trans. Amer. Inst. min. metall. Engrs. 175, 742–756 (1948).

The thermal expansivities (increase in length divided by original length) from 30 to 850 C in the iron-cobalt system decrease from the component ends of the system to a minimum in the region of atomic 50 %. The linear differential coefficients of thermal expansion at 200, 400, and 600 C behave in a similar fashion, but at 750 C the coefficients do not vary much with composition except for a minimum associated with the alpha-gamma phase change. The slowly cooled, well-ordered alloys in the region of 50% cobalt have lower expansivities than quenched, more randomly arranged alloys; well-ordered samples have larger specific volumes at room temperature.

Interatomic distances deviate positively from a simple linear variation with atomic per cent. These deviations, as well as the positive deviation of the average magnetic moment at saturation per atom and the negative deviation of the coefficient of thermal expansion from linear variations with composition are explained by the transfer of 3d electrons from cobalt to iron when the atoms are mixed in an alloy. R. L. Pigford, USA

780. **Paul J. Flory, Thermodynamics of crystallization in high polymers. I. Crystallization induced by stretching,** J. chem. Phys. 15, 397–408 (June 1947).

The methods of statistical mechanics are used to develop a theory of oriented crystallization in elongated polymers having three-dimensional network structures. In accordance with the second law of thermodynamics this theory leads to the conclusion that equilibrium crystallization decreases the tension in specimens held at a constant elongation. However, this is in contradiction with observations which indicate that the tension in highly elongated vulcanized rubber which has undergone crystallization during stretching, is greater than in another identically treated material in which no crystallization occurs at the same elongation.

As is pointed out in the paper, this contradiction is the consequence of the limitation of thermodynamical theories to states of equilibrium, which do not actually occur in this case because of the formation of crystallites during the stretching processes. Due to crystallization there is a rapid increase in the number of elastic elements capable of resisting the load, with a corresponding decrease in entropy, resulting in an increasingly steep slope of the stress-strain curve for crystallizing rubbers. In order to test this theory, the author proposes experiments in which stretching would be carried out at elevated temperatures, to prevent crystallization during the process, and to assure that the subsequent crystallization will take place at a fixed elongation, which would represent more satisfactorily the equilibrium state on which the present theoretical considerations are based.

M. Hetényi, USA

781. **M. Mooney, The thermodynamics of a strained elastomer. I. General analysis,** J. appl. Phys. 19, 434–444 (May 1948).

This article is entirely theoretical and is the first of a series of papers which will present theory and experiment in the treatment of the thermodynamics of strained elastomers. By subordinating volume-change effects to shape-change effects, and assuming that at any deformation the volume is a linear function of temperature and pressure, the basic partial differential equations of thermodynamics can be integrated and yield expressions for energy and entropy of deformation in terms of observed stresses and temperature changes in a deformed sample. Knowing the necessary form of the entropy and energy functions of deformation the labor of curve fitting to experimental data can be reduced.

For elastomers which crystallize when stretched, crystallinity is taken into account with the assumption that per cent crystallinity, like the volume, is a linear function of pressure and temperature. The theory can be made more exact by including second- or higher-order terms of temperature and pressure at any time and following the same procedure.

It is also shown that the results obtained by the author in a previous paper on the strain-work function for a large elastic deformation have a general validity for most cured elastomers when incorporated in the present thermodynamic analysis.

Steven Yurenka, USA

782. **L. E. Copeland, The thermodynamics of a strained elastomer. II. Compressibility,** J. appl. Phys. 19, 445–449 (May 1948).

The author measured by means of a special apparatus the adiabatic compressibility coefficients of a number of commercial cured elastomers under pressures up to 5000 psi. These experimental data are intended to supply the necessary parameters for a number of equations derived in the preceding paper of the series for the treatment of the thermodynamics of strained elastomers. Isothermal compressibilities and the internal pressure of the elastomer were calculated from the measured adiabatic compressibilities and other thermodynamic properties.

A preliminary experiment was reported which showed that the thermal expansion and volume compressibility of samples of Hevea gum decreased slightly with elongations of 200%.

Steven Yurenka, USA

783. **L. E. Copeland and M. Mooney, The thermodynamics of a strained elastomer. III. The thermal coefficient of modulus and the statistical theory of elasticity,** J. appl. Phys. 19, 450–455 (May 1948).

The authors have analyzed available thermoelastic data by means of thermodynamic equations derived in the first paper of this series in order accurately to determine the relative contributions of the energy and entropy of deformation in developing the retractive force. The constants of the equation were fitted to the data by the method of least squares, and the agreement between observed data and calculated curves was fairly good. These constants provided tests for the statistical mechanical theory of elasticity, and variance analyses show that several different elastomers depart from its predictions.

From a series of torsion-relaxation experiments of elastomers at small strains and varying temperatures it was determined that the stress is approximately proportional to the absolute temperature.

Steven Yurenka, USA

784. **F. Joder, The heat pump with some notes on its application to air-conditioning in land and marine service,** Trans. Inst. mar. Engrs. 60, 223–235 (Dec. 1948).

The author discusses various types of heat pumps after giving a summary of the well-known principles involved. The following types are reviewed: (a) heat pump for evaporating and boiling processes; (b) heat pump to extract heat from waste warm water from industrial processes; (c) heat pump for space heating. Conclusions are drawn for each group and it is recommended that, due to low coefficient of performance, the heat-pump installation for space heating should be given careful consideration from the economic standpoint.

A. D. Kafadar, USA

Heat Transfer; Diffusion

(See also Revs. 755, 759, 760)

785. **Max Jakob, Heat transfer. Vol. I,** New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1949, xxix + 758 pp., 247 figs. Cloth, 6 × 9.3 in., $12.

The hub of this book is the very active and pioneer work of the author and his co-workers first in Germany and more recently in America. In the main, the text is based on German heat-transfer literature of the past quarter of a century, with the inclusion of the ample experimental data accumulated in America in recent years. There is no doubt that this important treatise will be welcomed by student, research worker and engineer alike. The mathematical treatment of this subject is extremely lucid, and space is not wasted on rigid proofs for which adequate references are made.

The early part of the book is devoted to the formulation of the basic differential equations of conduction, convection and radiation. These are followed by a very detailed discussion of numerical values for the important thermal properties of solids and fluids. The remainder of the first volume is divided equally between heat-conduction problems in the steady and unsteady states and heat-convection problems with and without change of phase and constitution. The solution of heat-radiation problems

and the application of all three processes to practical fields are deferred to volume II.

The treatment of the solutions of the differential equations of heat conduction for various boundary conditions is very complete. The numerous methods described include the classical, graphical, conformal mapping, sources and sinks, Heaviside operators, Laplace transforms, relaxation, finite differences and electrical analogy; each method of course has its own field of application. Much attention is paid to the solution by numerical methods, and numerous worked-out examples are given in the text.

On the other hand the chapters on heat convection include only a few of the known theoretical solutions of free and forced convection in both laminar and turbulent flow. The important part of this section lies in the careful correlation of the experimental data and the derivation of formulas based on dimensional analysis. The important uses of optical methods, such as the shadowgraph and interferometer for measuring the heat transfer by free convection, are adequately described. (Some of the less amply treated topics in this section are: (a) forced convection about airfoils; (b) changes in heat-transfer coefficients with higher Mach number; (c) temperature distribution in pipes by the vorticity-transport theory; (d) temperature distribution in wakes and jets; (e) approximate solutions of problems where the change, with the temperature, of the thermal properties of the fluid is important.)

The subject matter of the first volume is concluded with a very complete treatment, based on both theory and experiment, of the similarity between the processes of heat transfer and diffusion, and the heat transfer associated with evaporation and condensation.

In appendixes the author has given a number of problems, without answers, for the use of students, and a bibliography which includes over 600 references. The book is happily free from major printing errors. Attention is drawn to equations 22-21 and 22-100 on pages 448 and 476, resp., which appear to contain misprints. G. M. Lilley, England

786. S. Paterson, The heating or cooling of a solid sphere in a well-stirred fluid, Proc. phys. Soc. Lond. 59, 50–58 (Jan. 1947).

The author analyzes mathematically the transient temperature distribution within a sphere which is heated or cooled by immersion in a finite quantity of well-stirred fluid. For the system as a whole there is neither loss nor gain of heat. An expression is also obtained for the uniform, but not constant, fluid temperature.

The first approach is by way of an infinite series along the lines adopted by W. Peddie and mentioned by H. S. Carslaw and J. C. Jaeger in their work, *Conduction of Heat in Solids*. It is noted that the rapidity of convergence of the series obtained varies markedly with the magnitude of the parameter $\tau = kta^{-2}$, in which k represents the diffusivity, and a the radius of the sphere, while t is the time. For low values of τ an excessive number of terms is required for satisfactory accuracy in numerical work.

The author then proceeds to use the repeated error function integrals of Hartree to obtain an alternative expression especially suited for application with low values of τ. Satisfactory agreement between the methods is demonstrated at $\tau = 0.1$.

Curves are presented which facilitate the application of the methods to specific problems. R. G. Boiten, Holland

787. Yasundo Takahashi, Heat or material transfer between porous solid and flowing fluid (in Japanese), J. Soc. mech. Engrs. Japan 50, no. 351, 376–377 (Nov. 1947).

It has been shown by the author that the problem of the temperatures of a flowing fluid and a porous solid (the wall of the pipe) when heat exchange takes place between them, is reduced under certain conditions to that of a cross-flow heat-exchanger [Kôgyô Zassi 79, p. 454 (1943)]; a graphical method for the latter problem has also been proposed [Trans. Soc. mech. Engrs. Japan 8, no. 30, part 2, p. 1 (1942)].

The author shows in the present article that his method can be applied to the percolation problem, and presents, as an example, a graphical method for obtaining the temperature field for the case of a porous solid (or particles in the flow) when heat-of-contact reaction is taken into account. Humio Tamaki, Japan

788. Yasusi Tanasawa, Yosiki Miyasaka, and Yotuo Yamada, Gas burners in a high-speed air stream (in Japanese), J. Soc. mech. Engrs. Japan 50, no. 345, 121–122 (Apr.–May 1947).

Characteristics of various types of gas burners in air stream are reported. When the burning gas is discharged simply through a cylindrical tube, the flame is easily blown out by an air stream. By attaching a diffusor of suitable dimension to the tube, the velocity at which the flame is blown out is greatly raised. It is also shown that, when suitable rotation is given to the burning gas when injecting it into the diffusor, disappearance of the flame is delayed still more. Humio Tamaki, Japan

789. Karl Wirtz, Kinetic theory of thermo-osmosis (in German), Z. Naturforsch. 3A, 380–386 (July 1948).

The ratio of pressure difference to temperature difference in thermo-osmosis is related to the heat of transfer [see E. D. Eastman, J. Amer. chem. Soc. 48, p. 1482 (1926); 50, p. 283 (1928)]. Only the effects at the surfaces of the permeable membrane are significant. In special cases the heat of transfer is directly related to the heat of solution of the diffusing fluid in the membrane. Serge Gratch, USA

Ballistics, Detonics (Explosions)

(See also Rev. 747)

790. Maurice Garnier, Vertical trajectories by the G.H.M. method (in French), Mémor. Artill. fr. 23, no. 1, 125–152 (1949).

This article is a supplement to two previously published articles on the same subject by the author in the Mémorial de l'Artillerie Française. In the earlier articles, published in 1930 and 1934, an approximate method (known as the G.H.M. method) for the computation of trajectories is developed, both for the general case and for vertical trajectories. In the present paper, the computation of vertical trajectories is discussed in greater detail. H. Polachek, USA

791. A.-L.-M. Gabeaud, On the efficiency of projectiles against soils and concretes (in French), Mémor. Artill. fr. 23, no. 1, 153–183 (1949).

In this paper formulas are developed for the penetration of projectiles into soil and concrete. Predictions are compared with the results of firing trials. Idealized models are used to calculate the behavior during penetration, which include the influence of friction and target-material inertia effects.

A formula previously given for armor penetration is modified for penetration into soil. The constants required for application to various soils are determined by firing trials.

The motion of a projectile penetrating concrete is studied in detail, using a resistance law based on integration of the pressure and friction tractions acting on the penetrating nose. The friction and penetration constants are determined by static tests. Oblique incidence is also analyzed.

An idealized analysis of explosive effects is also given, showing how crater and camouflet dimensions can be calculated from the explosive characteristics and dimensions, and its position in ground of specified strength and density. E. H. Lee, USA

792. **L. Don Leet, Vibrations from delay blasting,** Bull. seism. Soc. Amer. 39, 9–20 (Jan. 1949).

Results of a few recent experiments on delay blasting to reduce vibration are described. An earlier Bureau of Mines Bulletin (no. 442) has concluded that delay techniques would not be effective in reducing vibration by wave interference, because there is often no regular frequency. Nevertheless, the quoted results show favorable effects on both amplitude of vibration and type of breakage. A tentative explanation is given, based on conditions in the vicinity of the shot. R. Smelt, USA

Soil Mechanics, Seepage

(See also Rev. 722)

793. **G. A. Oosterholt, Approximate calculations of the stress distributions due to concentrated vertical loads** (in English), Proc. Sect. int. Conf. Soil Mech. Found. Engng. 1, 112–115 (1948).

The author presents an approximate method for computing the intensity of radial stresses in a material, due to concentrated vertical loads. It is assumed that the modulus of elasticity of the loaded mass increases linearly with depth and is dependent on the load to which the mass is subjected. The concentration factor for the intensity of radial stresses as thus computed is found to be considerably greater than that indicated by the Boussinesq solution. R. E. Fadum, USA

794. **Ryuma Kawamura, The motion of sand due to wind** (in Japanese), Science (Kagaku) 18, 500–506 (Nov. 1948).

The motion of sand due to wind can be classified into suspension, saltation and surface creep. When the wind velocity exceeds some critical value, the motion of the particles is accelerated and saltating motion begins. The author finds the relation between this critical wind velocity and the diameter of sand. Assuming the distribution of repulsive velocities of sand particles at the surface of sand mass to be vertical and of Maxwell's type, he obtains the law of distribution of the amount of sand flowing with the wind. He also obtains the relation between the amount of sand flow and the wind velocity. Further, he discusses the surface creep and the formation of sand ripples.

The author compares his theoretical results with those of Chepil's, Bagnold's and his own experiments. Qualitative agreement is very satisfactory. T. Mogami, Japan

795. **F. Sauvage de St. Marc, Flow in a porous medium, seepage under dams** (in French), Houille blanche 2, 126–134 (Mar.–Apr. 1947).

The paper deals with the computation of seepage under the foot of a dam if the finite thickness of the permeable layer is taken into account. For the simplest, schematized arrangements the solution is given with the aid of hyperbolic and elliptic functions as mapping functions; the results are evaluated and represented graphically. For more complicated dam designs a graphical solution is outlined but is not presented in detail.

Paul Neményi, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 659, 736, 738, 794)

796. **Rafael Davila Cuevas, General equations of variations of temperature in the atmosphere; secondary, dynamic and adiabatic processes in the variation** (in Spanish), Revista Ci. Lima 49, 247–267 (1947).

797. **H. Bondi, The growth of meteorological disturbances,** Proc. Cambridge Philos. Soc. 45, 92–98 (1949).

The motions of a gas slightly disturbed from equilibrium under the influence of gravity are considered. Heat conduction and viscosity are at first neglected. The well-known sharp distinction between slow large-scale (meteorological) and fast small-scale (acoustical) phenomena is confirmed by the mathematical analysis. Only the former motions are considered here, and the author confines himself to those disturbances which involve regions in which the percentual change of the temperature is small. For the stable case the result of the disturbance will be a slow oscillation, while in the unstable case if the lapse rate exceeds the adiabatic by 1%, the strength of the disturbance will be multiplied by 10,000 within $1^1/_2$ hr. If the effects of viscosity and heat conduction are considered in a semiempirical fashion, it can be concluded that because of the increase of the dissipating effects with decreasing magnitude of the disturbance, a limiting size must exist below which the disturbance cannot grow, even though the stratification is unstable. If the lapse rate exceeds the adiabatic by 1% this limit would be represented by the diameter 230 m. The effect of the earth's rotation is negligible.

Courtesy of Mathematical Reviews B. Haurwitz, USA

798. **Stanko Bilinski, Contribution to the dynamics of the cumulonimbus cloud** (in Croatian, with English summary), Hrvatsko Prirodoslovno Društvo, Glasnik Mat.-Fiz. Astr. Ser. II, 3, 29–51 (1948).

An explanation is given for the barograph trace during the passage of a cumulonimbus, viz., the sudden increase and subsequent fall of the pressure. It is first shown that a descending air current can originate in the region of heaviest precipitation of the cumulonimbus because friction of the air on the raindrops can overbalance the buoyancy of the warm ascending current. Then a possible field of flow in a cumulonimbus is derived, and it is shown that this field of motion leads to a pressure distribution with a pressure rise during the passage of the cloud.

Courtesy of Mathematical Reviews B. Haurwitz, USA

799. **S. V. Dobroklonskiĭ, Turbulent viscosity in the surface layer of the ocean and swell** (in Russian), Doklady Akad. Nauk SSSR (N.S.) 58, 1345–1348 (1947). Review delayed.

800. **V. B. Shtokman, Connections between the wind field, the complete current field and the average mass field in an inhomogeneous ocean** (in Russian), Doklady Akad. Nauk SSSR (N.S.) 59, 675–678 (1948). Review delayed.

801. **V. B. Shtokman, The influence of the profile of the bottom on the direction of the mean current excited by the wind or the field of a mass in an inhomogeneous ocean** (in Russian), Doklady Akad. Nauk SSSR (N.S.) 59, 889–892 (1948). Review delayed.

802. **Guy C. Omer, Jr., Differential-motion seismographs,** Bull. seism. Soc. Amer. 37, 157–215 (July 1947).

In this paper the author describes briefly the Benioff linear-strain seismograph and gives the general mathematical solution for this type of seismograph by a routine application of the Laplace-transformation method. Next, he indicates several

variations of the Benioff seismograph, and by mathematical analysis draws conclusions as to the advantages of each type considered. Finally the author considers the possibility of using optical seismographs built on the principle of the Michelson interferometer, and concludes that this type of seismograph would be both practical and useful. In conclusion the author points out that the differential-motion seismographs considered in the paper are at a disadvantage with respect to the pendular seismographs for routine seismology, but that they have advantages in special fields.

N. O. Myklestad, USA

Marine Engineering Problems

(See also Revs. 673, 766)

803. **M. Gertler, A method for converting the British C coefficient based on the Froude "O" values to an equivalent C coefficient based on the Schoenherr frictional formula,** David Taylor Model Basin Rep. no. R-657, 7 pp. (July 1948).

The author describes a semigraphical method of converting the circled-C coefficients, which are widely used for the nondimensional presentation of ship-resistance data, to an equivalent coefficient based on the newer, Schoenherr friction formulation. The conversion from one to the other coefficient is first precisely formulated and then presented in the form of a chart for ready use. An example illustrating its use is included.

Karl E. Schoenherr, USA

804. **L. Troost, Resistance and propulsion. The effect of shape of entrance on ship propulsion,** Shipbuilder 56, 311–318 (Apr. 1949).

The results of self-propulsion tests on the model of a Dutch Coaster, 137 ft long, propelled by a motor of 300 bhp, at 300 rpm are presented.

From model tests, in which the type of entrance only is changed it appears that at the design speed of 9.5 knots a normal form requires the minimum power with a wedge-shaped entrance 2% higher and a bulbous-bow entrance 6% higher. Other tests with propeller diameters $2^1/_4$%, 7% and 11% less than the open-water optimum, indicated that a propeller having a diameter 5% less than the open-water optimum performs most efficiently.

The effect of a small trip wire acting as a turbulence device located ahead of the model was shown to be greater for the normal and the wedge form than for the bulbous-bow form.

F. E. Reed, USA

805. **F. B. Bull and J. F. Baker, Strength of ships—The measurement and recording of the forces acting on a ship at sea,** Shipbuilder 56, 299–306 (Apr. 1949).

In order to determine the appropriate loads to apply in an experimental static-load comparison of stresses in a welded ship and a riveted ship, dynamic loads were measured on the welded ship in the open sea under normal operating conditions. Water-pressure gages and accelerometers were located at 12 stations along the length of the ship. The height of the wave surface on the side of the ship was measured by electrical wave-profile indicators. Strain gages were also used for checking purposes. All gages were electrically operated and connected to one central control room, where the indicators and meters were mounted on a large panel and photographed every half second by a special camera.

Over a period of 17 months, 40,000 frames of film, each yielding about 120 individual readings, were obtained for open-sea conditions. In the present paper only a few preliminary results are presented. Very good agreement is found between stresses calculated from the measured loads and strain-gage records. A few cases of severe impact indicated high localized water pressures such as are known to occur in seaplane-landing impacts.

Stanley U. Benscoter, USA